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On Transient Solutions of the "Baffled Piston" Problem

F. Oberhettinger [1]

(September 14, 1960)

The acoustic field produced by the movement of a piston in an infinite rigid wall for arbitrary time dependency of the motion is given.

1. Introduction

The case of the time harmonic movement of a piston membrane in an infinite rigid wall ("baffle") can easily be generalized to the case when the motion of the piston is not periodic but an arbitrary function of time. Such transient solutions have become of considerable interest in recent times (for a detailed treatment of the propagation of such sound pulses, see [5][2]). The procedure for the case treated here is the same as used elsewhere [7], i.e., the Green's function for the exponential decay case (modified wave equation $\Delta U-\gamma^2 U=0$, $\gamma=ik$) is used to obtain the solution for the pulse problem. The acoustic field (velocity potential) for the time harmonic movement of the piston include representations given by Bouwkamp [1], King [6], and Wells and Leitner [9]. The first of these contributions gives the solution in the form of a series expression while the second and third involve integral representations that are obtained using integral transform methods (Hankel transform [4, p. 73] and Lebedev transform [4, p. 75] respectively). These representations can be used to treat the general case of an arbitrary movement of the piston. In view of the method to be employed here, such representations should be used for which the inverse Laplace transform of the velocity potential with respect to the purely imaginary wave parameter $\gamma=ik$ can be given. Such an expression can be obtained in a direct way by regarding each point of the moving disk as an acoustic point source and integrating over all points of the disk.

2. Exponential Decay and Time Harmonic Solution

The piston is represented by an infinitely thin circular disk of radius a located in the x,y-plane with its center at the origin of a three-dimensional Cartesian system of coordinates. The remaining part of the x,y-plane consists of a rigid wall. The movement of the disk is time harmonic of the form

$$v_z=e^{i\omega t} \tag{1}$$

where v_z is the velocity of the disk.

Each point Q of the disk can be considered as a point source with the velocity potential

$$\Phi=-\eta\,\frac{e^{-ikr}}{4\pi r}\,e^{i\omega t}\rho' d\varphi' d\rho', \quad r=PQ, \tag{2}$$

($\omega=kc$, c is the velocity of sound), which represents Green's function of the wave equation in free space (see fig. 1).

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[1] Oregon State College, Department of Mathematics (invited paper).

[2] Figures in brackets indicate the literature references at the end of this paper.

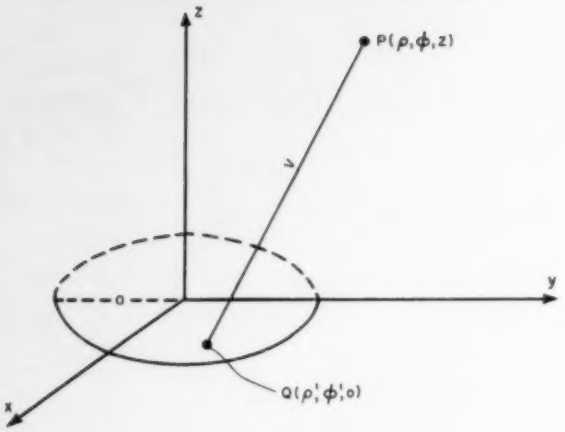

FIGURE 1.

Upon substituting

$$k=-i\gamma, \quad \omega=-i\gamma c, \tag{3}$$

one obtains, after isolation of the time-dependent factor $e^{\gamma c t}$ for the free-space Green's function of

$$\Delta u-\gamma^2 u=0, \tag{4}$$

the expression

$$u=-\eta \frac{e^{-\gamma r}}{4\pi r} \rho' d\varphi' d\rho', \tag{5}$$

$$r=[\rho^2+\rho'^2-2\rho\rho' \cos(\varphi-\varphi')+z^2]^{1/2}. \tag{6}$$

The transition from k to γ by (3) amounts to the transition from a wave problem to an exponential decay problem. At first the exponential decay problem for positive real γ is solved and the solution for the wave problem can be obtained by returning from γ to k. This, however, is not necessary here, since the exponential decay solution will be used in order to solve the problem of an arbitrary moving disk. If the expression in (5) is integrated over the whole disk, one obtains—observing (6):

$$U=-\frac{\eta}{4\pi}\int_{\varphi'=0}^{2\pi}\int_{\rho'=0}^{a}[\rho^2+\rho'^2-2\rho\rho' \cos(\varphi-\varphi')+z^2]^{-1/2} \cdot \exp\{[\rho^2+\rho'^2-2\rho\rho' \cos(\varphi-\varphi')+z^2]^{1/2}\} \rho' d\rho' d\varphi'. \tag{7}$$

Here η is the "density distribution" over the disk. Since the disk moves as a whole, this property is a constant; it is determined by the fact that according to (1)

$$v_z=-\left(\frac{\partial U}{\partial z}\right)_{z=0}=\begin{cases}1, \text{ for } \rho<a,\\ 0, \text{ for } \rho>a.\end{cases} \tag{8}$$

This function U as defined in (7) satisfies

(a) $$\Delta U-\gamma^2 U=0,$$

(b) $$\frac{\partial U}{\partial z}=-1 \text{ for } \rho<a, \qquad \frac{\partial U}{\partial z}=0 \text{ for } \rho>a \qquad \text{at } z=0,$$

(c) $$\lim U\to 0 \text{ when } \rho \text{ or } |z|\to\infty.$$

In order to perform the integration in (7), one replaces the integrand in (7) using the well-known formula [3, vol. 1, p. 191, 9],

$$\int_\gamma^\infty J_0[\lambda(t^2-\gamma^2)^{1/2}]e^{-zt}dt=(z^2+\lambda^2)^{-1/2}e^{-\gamma(z^2+\lambda^2)^{1/2}},\quad z\geqq 0$$

or, upon substituting

$$t^2-\gamma^2=\tau^2,\ t^2=\tau+\gamma^2$$

$$(z^2+\lambda^2)^{-1/2}e^{-\gamma(z^2+\lambda^2)^{1/2}}=\int_0^\infty J_0(\lambda\tau)\tau(\tau^2+\gamma^2)^{-1/2}e^{-z(\tau^2+\gamma^2)^{1/2}}d\tau \tag{9}$$

If this formula is used to replace the integrand in (7), one obtains upon interchanging the order of integration

$$U=-\frac{\eta}{4\pi}\int_0^\infty \tau(\tau^2+\gamma^2)^{-1/2}e^{-z(\tau^2+\gamma^2)^{1/2}}\times\left\{\int_0^a\int_0^{2\pi}J_0(\tau[\rho^2+\rho'^2-2\rho\rho'\cos(\varphi-\varphi')]^{1/2})\rho'd\rho'd\varphi'\right\}d\tau. \tag{10}$$

To evaluate the integral in (10), one uses the addition theorem for the Bessel functions [4, p. 101]

$$J_0\{\tau[\rho^2+\rho'^2-2\rho\rho'\cos(\varphi-\varphi')]^{1/2}\}=\sum_{m=0}^\infty \epsilon_m J_m(\tau\rho)J_m(\tau\rho')\cos[m(\varphi-\varphi')],$$

$$\epsilon_0=1,\quad \epsilon_m=2,\quad m=1,2,3,\ldots.$$

Integrating with respect to φ' and observing that only the term $m=0$ gives a contribution, yields

$$U=-\frac{1}{2}\eta\int_0^\infty \tau(\tau^2+\gamma^2)^{-1/2}e^{-z(\tau^2+\gamma^2)^{1/2}}J_0(\tau\rho)\left\{\int_0^a\rho' J_0(\tau\rho')d\rho'\right\}d\tau.$$

Finally, using the result [4, p. 45, 1]

$$\int_0^a J_0(\tau\rho')\rho'd\rho'=\frac{a}{\tau}J_1(\tau a),$$

one obtains

$$U=-\frac{1}{2}a\eta\int_0^\infty(\tau^2+\gamma^2)^{-1/2}e^{-z(\tau^2+\gamma^2)^{1/2}}J_0(\tau\rho)J_1(\tau a)d\tau. \tag{11}$$

Now, according to (1)

$$-\left(\frac{\partial U}{\partial z}\right)_{z=0}=1,\quad \rho<a;\qquad -\left(\frac{\partial U}{\partial z}\right)_{z=0}=0,\quad \rho>a.$$

therefore by [3, vol. 2, p. 14],

$$-\left(\frac{\partial U}{\partial z}\right)_{z=0}=-\frac{1}{2}a\eta\begin{cases}a^{-1}, & \rho<a,\\ 0, & \rho>a.\end{cases}$$

Hence

$$\eta=-2.$$

The final representation becomes therefore

$$U=a\int_0^\infty(\tau^2+\gamma^2)^{-1/2}e^{-z(\tau^2+\gamma^2)^{1/2}}J_0(\tau\rho)J_1(\tau a)d\tau, \tag{12}$$
$$z\geqq 0.$$

Clearly U satisfies the conditions a, b, c, stated following (8). The restriction $z\geqq 0$ in (12) means no loss of generality, since it is sufficient to consider the upper half space. If one wishes

to return to the wave problem one has to replace γ by ik according to (3) and one arrives in this case at King's solution [6].

Approximate evaluations of King's and related integrals for various cases have been carried out [2, 6, 8].

3. Transient Solutions

The expression (12) is now used to investigate the corresponding pulse problem. Let the movement of the piston be represented by a pulse starting at $t=0$; let the pulse function be $g(t)$, and $g(t)=0$ for $t<0$. Under the assumption that the pulse function $g(t)$ is such that it admits its representation by Laplace's integral formula

$$g(t)=(2\pi i)^{-1}\int_{c-i\infty}^{c+i\infty}\left[\int_0^\infty g(\tau)e^{-\gamma\tau}d\tau\right]e^{\gamma t}d\gamma, \tag{13}$$

one can represent the expression for the acoustic field generated by the pulse in the form

$$\Phi(t)=(2\pi i)^{-1}\int_{c-i\infty}^{c+i\infty}\left[\int_0^\infty g(\tau)e^{-\gamma\tau}d\tau\right]Ue^{\gamma t}d\gamma \tag{14}$$

where U is the representation (12) for the "exponential decay" field. (The velocity of sound has been chosen to be unity.)

If the order of integration can be interchanged one obtains

$$\Phi(t)=\int_0^\infty g(\tau)\left[(2\pi i)^{-1}\int_{c-i\infty}^{c+i\infty}Ue^{\gamma(t-\tau)}d\gamma\right]d\tau. \tag{15}$$

The integral in the bracket is the inverse Laplace transform of the "exponential decay" solution with respect to the wave parameter γ. If this is known, then the velocity potential $\Phi(t)$ of the acoustic field for the case of an arbitrary movement of the piston can be obtained by a further integration involving the pulse function $g(t)$ according to (15). The inverse Laplace transform of the integrand in (12) with regard to γ is known [3, vol. 1, p. 248]:

$$(2\pi i)^{-1}\int_{c-i\infty}^{c+i\infty}(\tau^2+\gamma^2)^{-\frac{1}{2}}e^{-z(\tau^2+\gamma^2)^{1/2}}e^{\gamma t}d\gamma=\begin{cases}J_0[\tau(t^2-z^2)^{\frac{1}{2}}]\\0\end{cases} \tag{16}$$

according as $t>z$ or $t<z$, respectively.

The acoustic velocity potential due to a pulse function $g(t)$ ("Dirac" pulse movement of the piston at $t=0$) is by (14)

$$\Phi_D(t)=2\pi i)^{-1}\int_{c-i\infty}^{c+i\infty}U_o e^{\gamma t}d\gamma. \tag{17}$$

Hence, (15) can be written as

$$\Phi(t)=\int_0^\infty g(\tau)\Phi D(t-\tau)\,d\tau. \tag{18}$$

Therefore, by (12) and (16)

$$\Phi_D(t)=\begin{cases}0, & t<z\\ a\int_0^\infty J_1(\tau a)J_0(\tau\rho)J_0[\tau(t^2-z^2)^{\frac{1}{2}}]d\tau, & t>z.\end{cases} \tag{19}$$

The integral is known. One has [3, vol. 2, p. 21]:

$$\int_0^\infty J_0(bx)J_0(cx)J_1(xy)dx=\begin{cases}0, & 0<y<|b-c|\\ (\pi y)^{-1}\text{ arc }\cos\left(\dfrac{b^2+c^2-y^2}{2bc}\right), & |b-c|<y<b+c\\ y^{-1}, & y>b+c\end{cases} \tag{20}$$

In order to evaluate (19) identify y with a, b with $(t^2-z^2)^{1/2}$ and c with ρ. Two different cases have to be distinguished:

Case A. $\quad a>\rho,$

Case B. $\quad a<\rho,$

i.e., the two possible different cases where the projection S of the point of observation P into the x,y-plane lies either inside or outside the disk are treated separately (fig. 2 and fig. 3 respectively).

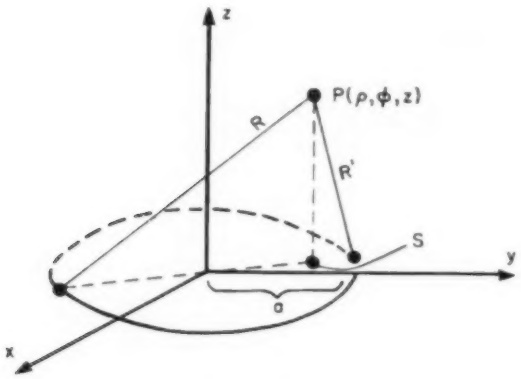

FIGURE 2.

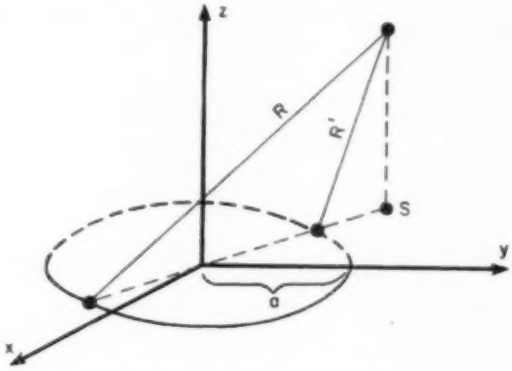

FIGURE 3.

The largest and the smallest distances of a point on the circumference of the disk from the point of observation P are denoted by R and R' respectively.

$$R=[z^2+(a+\rho)^2]^{1/2}, \qquad R'=[z^2+(a-\rho)^2]^{1/2}. \tag{21}$$

One obtains then for $\Phi_D(t)$ by (19) under consideration of (20) and (21):
For the case A, $a>\rho$:

$$\begin{aligned}
\Phi_D(t)&=0, && t<z\\
&=1, && z<t<R'\\
&=\pi^{-1}\arccos\left[\frac{t^2-z^2+\rho^2-a^2}{2\rho(t^2-z^2)^{1/2}}\right], && R'<t<R\\
&=0, && t>R.
\end{aligned} \tag{22}$$

For the case B, $a<\rho$:

$$\begin{aligned}
\Phi_D(t)&=0, && t<R'\\
&=\pi^{-1}\arccos\left[\frac{t^2-z^2+\rho^2-a^2}{2\rho(t^2-z^2)^{1/2}}\right], && R'<t<R\\
&=0, && t>R.
\end{aligned} \tag{23}$$

The physical interpretation of these results is obvious. The parts in (22) and (23) that are different from zero represent the transient acoustic field at a point P due to a "Dirac" pulse movement of the piston for a time which lies between the smallest and the largest distance of the point of observation P from the disk. Outside of this the field is zero. (Note that the velocity of sound was assumed to be unity.) The "Dirac" pulse solutions (22) and (23) can now be used to construct the acoustic field in case of an arbitrary time dependency $g(t)$ of the movement of the piston according to (18). One obtains immediately:

For the case A, $a>\rho$:

$$\Phi(t)=\pi^{-1}\int_{t-R}^{t-R'} g(\tau)\ \arccos\left\{\frac{(t-\tau)^2+\rho^2-a^2-z^2}{2\rho[(t-\tau)^2-z^2]^{1/2}}\right\}d\tau+\int_{t-R'}^{t-z} g(\tau)d\tau. \tag{24}$$

For the case B, $a<\rho$:

$$\Phi(t)=\pi^{-1}\int_{t-R}^{t-R'} g(\tau)\ \arccos\left\{\frac{(t-\tau)^2+\rho^2-a^2-z^2}{2\rho[(t-\tau)^2-z^2]^{1/2}}\right\}d\tau. \tag{25}$$

The case $\rho=0$ (point of observation on the axis of the disk) reduces to

$$\Phi(t)=\int_{t-(a^2+z^2)^{1/2}}^{t-z} g(\tau)d\tau. \tag{26}$$

It has to be remembered that $g(t)=0$ for $t<0$. Therefore no possibly negative part of the interval of integration in (24) to (26) gives a contribution. Furthermore, the time parameter t in the above formulas has to be replaced by ct, where c is the velocity of sound.

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Special Types of Partitioned Matrices

Emilie V. Haynsworth*

(September 14, 1960)

This paper extends the results of two previous papers on partitioned matrices. General reduction formulas are given for partitioned matrices A of order np satisfying $A(X\times I_p)=(X\times I_p)B$, where B is a matrix of order rp and $X\times I_r$ represents the direct product of an $n\times r$ matrix X of rank r with the identity matrix of order p. These formulas are related to the formulas given in the previous papers for partitioned matrices satisfying $A(I_p\times X)=(I_p\times X)B$.

1. Introduction

In a recent paper L. S. Goddard and H. Schneider [4][1] discussed the relationship between matrices A and B, of orders n and m respectively, which satisfy

$$AX=XB \tag{1}$$

for some $n\times m$ matrix, X, of rank $r>0$.

The results of Goddard and Schneider were generalized in a paper by the author [5] for partitioned matrices A and B, in which corresponding submatrices (or blocks), A_{ij} and B_{ij}, satisfy an analogous relationship to that given by (1); i.e.,

$$A_{ij}X_j=X_iB_{ij}, \qquad (i,j=1, \ldots ,p) \tag{2}$$

where A_{ij} is $n_i\times n_j$, X_i is $n_i\times m_i$ of rank r_i, and B_{ij} is $m_i\times m_j$.

If, in (2), each A_{ij} is square, of order n, B_{ij} is square of order r, and all X_i are equal to an $n\times r$ matrix, X, of rank r, then A is of order pn, B of order pr, and (2) can be written

$$A(I_p\times X)=(I_p\times X)B, \tag{3}$$

where $(I_p\times X)$ is a $pn\times pr$ matrix which is the direct product of the identity matrix of order p with the matrix X; i.e.,

$$I_p\times X=\begin{pmatrix} X & & & \\ & X & & \\ & & \ddots & \\ & & & X \end{pmatrix}$$

(The general definition for direct products is given in section 3.)

It was shown by the author in [6] that, given a matrix A satisfying (3) for some matrices B and X, there exists a matrix P, which can be constructed from X, such that

$$P^{-1}AP=\begin{pmatrix} B & * \\ 0 & C \end{pmatrix}, \tag{4}$$

so that pr of the roots of A are the roots of B. Thus, if $r<n$, or of B is reduced, (4) gives a reduction formula for A. Naturally a matrix satisfying (1) satisfies (3) with $p=1$.

In theorem 2 of this paper a similar reduction formula to those for matrices satisfying (1) and (3) is given for partitioned matrices A of order pn satisfying

$$A(X\times I_p)=(X\times I_p)B, \tag{5}$$

for some $n\times r$ matrix X of rank r, and the corresponding partitioned matrix B of order pr. The relationship between matrices satisfying (3) and (5) is discussed more fully in the next section.

Other reduction formulas for partitioned matrices are contained in the results of B. Friedman [3], S. N. Afriat [1], and J. Williamson [10]. These formulas will be shown to be special cases of theorem 2.

The reduction formulas given in section 5 are applied in section 7 to certain special partitioned matrices. In particular, a 9×9 matrix, involving an arbitrary parameter β, which arose in a problem on lattice points at the Bureau of Standards, is reduced by these formulas to a set of matrices of order 4, 2, 2, and 1 and from these reduced matrices all eigenvalues are found as functions of β.

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[1] Figures in brackets indicate the literature references at the end of this paper.

2. Inner- and Outer-Related Partitioned Matrices

The formulas (1), (3), and (5) may be related by the matrix X. For instance, if the matrix A in (1) is a circulant matrix,

$$A=\begin{pmatrix} a_0 & a_1 & \cdots & a_{n-2} & a_{n-1} \\ a_{n-1} & a_0 & \cdots & a_{n-3} & a_{n-2} \\ & & \cdots & & \\ a_1 & a_2 & \cdots & a_{n-1} & a_0 \end{pmatrix}, \tag{6}$$

and X is the matrix of its characteristic vectors, then the related matrix satisfying (3) will have each of its submatrices a circulant of order n, and the related matrix satisfying (5) will be in circulant form; i.e.,

$$A=\begin{pmatrix} A_0 & A_1 & \cdots & A_{n-1} \\ A_{n-1} & A_0 & \cdots & A_{n-2} \\ & & \cdots & \\ A_1 & A_2 & \cdots & A_0 \end{pmatrix}. \tag{7}$$

A matrix satisfying both (3) and (5) would be one which has the form (7), where each submatrix A_i has the form (6). D. E. Rutherford [9] has given reduction formulas for matrices of this type as well as for matrices related to tridiagonal matrices.

We will call matrices satisfying (3) "inner-related" partitioned matrices, or i.r.p. matrices, those satisfying (5) "outer-related," or o.r.p. matrices, and matrices satisfying both conditions "doubly-related," or d.r.p. matrices.

We shall show that dual reduction formulas exist for o.r.p. and i.r.p. matrices and indeed that an o.r.p. matrix can be put into i.r.p. form by a simultaneous permutation of rows and columns.

Since the formulas (3) and (5) are stated in terms of direct products, the general definition and three of the properties of direct products are given in section 3.

3. Direct Products of Matrices

W. E. Roth [8] derives some interesting results on direct product matrices and gives the general definition for rectangular matrices, together with some of the properties. The three properties needed for our results are given after the definition below. Most of the properties of direct products can also be found in MacDuffee [7], although he deals only with square matrices.

The direct product of an $m\times n$ matrix, A, and a $p\times q$ matrix, B, in that order, is an $mp\times nq$ matrix defined as follows:

$$A\times B=\begin{pmatrix} a_{11}B & a_{12}B & \cdots & a_{1n}B \\ a_{21}B & a_{22}B & \cdots & a_{2n}B \\ & & \cdots & \\ a_{m1}B & a_{m2}B & \cdots & a_{mn}B \end{pmatrix}.$$

Three properties of direct products:

1. If the products AC and BD exist,
$$(A\times B)(C\times D)=AC\times BD.$$
2. If A and B are nonsingular,
$$(A\times B)^{-1}=(A^{-1}\times B^{-1}).$$
3. If A is an $m\times n$ matrix and B is a $p\times q$ matrix, $A\times B=P(B\times A)Q$, where P and Q are obtained from the identity matrices I_{mp} and I_{nq} by simultaneous permutation of rows and columns.

4. Similarity Between O.R.P. and I.R.P. Matrices

THEOREM 1: *If A is an outer-related partitioned matrix of order pn, with square blocks of order p, satisfying (5), then A can be transformed by simultaneous permutation of rows and columns into an inner-related partitioned matrix, $\tilde{A}$, with blocks of order n, satisfying (3).*

PROOF. Suppose A satisfies (5). Using property 3 of direct products, given above, there exist nonsingular permutation matrices, P and Q, such that

$$X\times I_p=P(I_p\times X)Q. \tag{8}$$

Then, from (8) and (5),

$$A[P(I_p\times X)Q]=[P(I_p\times X)Q]B,$$

or

$$(P^{-1}AP)(I_p\times X)=(I_p\times X)(QBQ^{-1}).$$

So, if we let

$$\tilde{A}=P^{-1}AP, \qquad \tilde{B}=QBQ^{-1},$$

the relationship (3) holds for $\tilde{A}$ and $\tilde{B}$, and, since P is a permutation matrix, $\tilde{A}$ is obtained from A by simultaneous permutation of rows and columns.

From this theorem we can reduce any o.r.p. matrix, if we know the reduction formula for the i.r.p. matrix, by first permuting the rows and columns and then using the i.r.p. reduction formula. However, it is usually much simpler to use the direct reduction formula for o.r.p. matrices which is given in the next section.

5. A General Reduction Formula

THEOREM 2: *Suppose we have a matrix A of order n satisfying (1), or a matrix of order pn satisfying (3) or (5) for some corresponding matrix B, where X is a given $n\times r$ matrix of rank r. Then in each case there exists a matrix P, depending upon X, such that (4) holds, so that B contains r (or pr) of the roots of A.*

The only new result here is the formula for a matrix satisfying (5), as the formula for a matrix satisfying (1) was given by Coddard and Schneider [4], and the one for a matrix satisfying (3) was given by the author in [6]. The three transformation matrices are given below, however, in order to show the relations among them, but the proof is not given in detail as it appears in the other papers.

PROOF. Since X is an $n\times r$ matrix of rank r, we will assume

$$X=\begin{pmatrix}X_1\\X_2\end{pmatrix}, \tag{9}$$

where X_1 is a nonsingular $(r\times r)$ matrix, for if a permutation of the rows of X is necessary to achieve this, it can be matched by a simultaneous permutation of the rows and columns of A; i.e., a permutation of the rows of X is equivalent to left multiplication by a nonsingular matrix Q, but a matrix A which satisfies (1) also satisfies

$$QAQ^{-1}(QX)=(QX)B. \tag{10}$$

In each case below we will use the matrix R, which we construct as follows: The first r columns of R are those of the $n\times r$ matrix X, which we assume to have the form (9). The remaining $n-r$ columns consist of an $r\times(n-r)$ block of zeros and the identity matrix of order $n-r$, i.e.

$$R=\begin{pmatrix}X_1 & 0\\X_2 & I_{n-r}\end{pmatrix}. \tag{11}$$

Then

$$R^{-1}=\begin{pmatrix}X_1^{-1} & 0\\-X_2X_1^{-1} & I_{n-r}\end{pmatrix}. \tag{12}$$

Using R we can construct the transformation matrices for nonpartitioned matrices satisfying (1), or partitioned matrices satisfying (3) or (5) or both.

Case 1. Non-partitioned matrices

Suppose A is an $n\times n$ matrix satisfying (1) where X is an $n\times r$ matrix of rank r and B is an $r\times r$ matrix. It was shown by the author in [6] that (4) holds for the matrix A if we let $P=R$.

Case 2. I.R.P. matrices

If A is a partitioned matrix of order np, with blocks of order n, satisfying (3), A also satisfies (1) where the matrix $I_p\times X$ replaces the matrix X. In this case it was shown by the author in [6] that, if we let $P=I_p\times R$, then

$$P^{-1}AP=\tilde{A},$$

where $\tilde{A}$ can be put into the reduced form on the right side of (4) by a simultaneous permutation of rows and columns. In this case B is of order pr.

Case 3. O.R.P. matrices

If A is a partitioned matrix of order np, with blocks of order p, satisfying (5), then A also satisfies (1) with X replaced by

$$(X\times I_p)=\begin{pmatrix}X_1\times I_p\\X_2\times I_p\end{pmatrix},$$

where, by property 2 of direct products, since X_1 is nonsingular, $X_1\times I_p$ is nonsingular. Thus, if we let

$$P=R\times I_p=\begin{pmatrix}X_1\times I_p & 0\\X_2\times I_p & I_{n-r}\times I_p\end{pmatrix}, \tag{13}$$

(4) will hold in this case also, and B will again be of order pr.

Case 4. D.R.P. matrices

If A is a matrix of order np, with blocks of order n, and satisfies both (3) and (5), i.e.

$$A\,(X\times I_n)=(X\times I_n)\,B,$$

and

$$A\,(I_p\times Y)=(I_p\times Y)\,C,$$

where X is $p\times r$ and Y is $n\times s$, then A can be reduced twice. For, by case 3, if we let $P_1=R_1\times I_n$, where R_1 is the $p\times p$ matrix given in case 3, we have

$$P_1^{-1}AP_1=\begin{pmatrix}B & *\\0 & D\end{pmatrix},$$

where B is of order nr and D is of order $n(p-r)$.

It is clear that premultiplication and/or postmultiplication of a partitioned matrix A, with blocks of order n, by a matrix whose blocks are scalar multiples of the identity matrix of order n produces a matrix whose blocks are linear combinations of the blocks of A.

Also, if any set of matrices has the property that

$$A_iX=XB_i, \qquad (i=1,\ldots,t)$$

then

$$\sum\,(c_i\,A_i\,X)=\sum\,(c_i\,X\,B_i),$$

or

$$(\sum c_i\,A_i)\,X=X(\sum c_i\,B_i),$$

so that any linear combination of the set satisfies an equation of the form (1) with the same matrix X.

Hence the matrices B and D satisfy

$$B(I_r \times Y) = (I_r \times Y)E,$$
$$D(I_{p-r} \times Y) = (I_{p-r} \times Y)G,$$

where E is of order rs and G is of order $s(p-r)$. Then, by case 2, there exist matrices P_B and P_D such that

$$P_B^{-1} B P_B = \begin{pmatrix} E & * \\ 0 & F \end{pmatrix},$$

$$P_D^{-1} D P_D = \begin{pmatrix} G & * \\ 0 & H \end{pmatrix}.$$

Thus, the roots of A are those of the matrices E, F, G, and H, of order rs, $r(n-s)$, $s(p-r)$, and $(n-s)\cdot(p-r)$, respectively.

It would be easier in this case however, to multiply the two transformation matrices $R_1 \times I_n$ and $I_p \times R_2$, where R_2 is the $n \times n$ transformation matrix corresponding to the matrix in case 2, and obtain, by property 1 of direct products, the single transformation matrix, $P = R_1 \times R_2$, which could be used to put A into the reduced form,

$$P^{-1} A P = \begin{pmatrix} E & * & * & * \\ 0 & F & * & * \\ 0 & 0 & G & * \\ 0 & 0 & 0 & H \end{pmatrix}.$$

For example suppose

$$S = \begin{pmatrix} A & B \\ C & D \end{pmatrix},$$

where A, B, C, and D are 3×3 stochastic matrices, i.e.

$$\sum_{j=1}^{3} a_{ij} = a,$$

$$\sum_{j=1}^{3} b_{ij} = b,$$

$$\sum_{j=1}^{3} c_{ij} = c,$$

$$\sum_{j=1}^{3} d_{ij} = d, \qquad (i = 1, 2, 3)$$

then S is an i.r.p. stochastic matrix as discussed in reference [6].

If we have also

$$A + B = C + D,$$

then S is a d.r.p. stochastic matrix.

Since a stochastic matrix with row-sum s has one root s with corresponding eigenvector $(1, 1, \ldots, 1)$ (cf. A. Brauer, [2]), the matrices R_1 and R_2 would be

$$R_1 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \qquad R_2 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$

Then

$$P = R_1 \times R_2 = \begin{pmatrix} R_2 & 0 \\ R_2 & R_2 \end{pmatrix},$$

and

$$P^{-1} = R_1^{-1} \times R_2^{-1} = \begin{pmatrix} R_2^{-1} & 0 \\ -R_2^{-1} & R_2^{-1} \end{pmatrix}.$$

So

$$P^{-1} S P = \begin{pmatrix} R_2^{-1}(A+B)R_2 & R_2^{-1} B R_2 \\ 0 & R_2^{-1}(D-B)R_2 \end{pmatrix}$$

$$= \begin{pmatrix} a+b & * & b & * \\ 0 & A_2 + B_2 & 0 & B_2 \\ 0 & 0 & d-b & * \\ 0 & 0 & 0 & D_2 - B_2 \end{pmatrix},$$

where

$$A_2 = \begin{pmatrix} a_{22} - a_{12} & a_{23} - a_{13} \\ a_{32} - a_{12} & a_{33} - a_{13} \end{pmatrix},$$

and B_2 and D_2 have similar forms.

Another example using i.r.p. and o.r.p. transformation matrices is given in section 7.

In each of the above cases, if $r = n$, the matrix P is much simpler; i.e., for 1, $P = X$, for 2, $P = I_P \times X$, for 3, $P = X \times I_p$ and for 4 if $s = p$, $P = X \times Y$.

6. Theorems by Williamson, Afriat, and Friedman

It was shown by the author in [6] that Williamson's theorem [10] on partitioned matrices follows from theorem 2, part 2, (theorem 1 in [6]), where $r = n$ and the matrices B_{ij} are triangular.

S. N. Afriat [1] gives a general discussion with a number of theorems on the determinant and characteristic roots of partitioned matrices, $A = (A_{ij})$, in which the submatrices A_{ij} all commute. One of his results is a generalization of Williamson's theorem and shows that if the roots of the commutative matrices A_{ij} are $\lambda_{ij}^{(k)}$ $(k = 1, \ldots, n;\ i, j = 1, \ldots, p)$ then the roots of the partitioned matrix $A = (A_{ij})$ are roots of the matrices $\lambda_k = (\lambda_{ij}^{(k)})$. Since, as Afriat proves, commutative matrices can be simultaneously

reduced to triangular form by the same unitary transformation, it is clear that $A=(A_{ij})$ satisfies (3), where X is the unitary matrix in question, and the corresponding matrices, B_{ij}, are triangular of order n. So Afriat's result on the characteristic roots of A also follows from theorem 2.

Theorem 2 also provides a new proof for the following theorems by Friedman [3].

THEOREM I. *Let $A_1, A_2, \ldots, A_t$ be n-dimensional square matrices and $B_1, B_2, \ldots, B_t$ be p-dimensional square matrices. Suppose that $A_1, A_2, \ldots, A_t$ have a common eigenvector, ζ, and the corresponding eigenvalues are $\lambda_1, \lambda_2, \ldots, \lambda_t$ respectively. Then p of the eigenvalues of*

$$C=B_1\times A_1+B_2\times A_2+\ldots+B_t\times A_t \qquad (14)$$

will be eigenvalues of the matrix

$$D=\lambda_1 B_1+\lambda_2 B_2+\ldots+\lambda_t B_t.$$

As Friedman points out, this is a generalization of the well-known theorem that the eigenvalues of $B\times A$ are the products, $\lambda_i\mu_j$ $(i=1, \ldots, n;\ j=1, \ldots, p)$, where λ_i are the roots of A and μ_j are the roots of B.

THEOREM II. *Let C be the matrix considered in theorem 1. Suppose the ring generated by the matrices $A_1, \ldots, A_t$ has an r-dimensional representation $(r<n)$ in which the matrix A_k is represented by M_k. Then pr of the eigenvalues of C are eigenvalues of the matrix*

$$D=B_1\times M_1+B_2\times M_2+\ldots+B_t\times M_t. \qquad (15)$$

Since theorem II contains theorem I as a special case with $r=1$, we will only prove theorem II.

PROOF. Friedman shows in his proof that the statement, "A_k is represented by M_k," is equivalent to

$$A_k X=XM_k, \qquad (k=1, \ldots, t) \qquad (16)$$

where the columns of the matrix X are the basis vectors for the subspace.

Let the elements of the matrix B_k be $b_{ij}^{(k)}$ $(i,j=1, \ldots, p;\ k=1, \ldots, t)$.

Then from (14), C is a partitioned matrix of order pn with blocks of order n,

$$C_{ij}=\sum_{k=1}^{t} b_{ij}^{(k)} A_k. \qquad (i,j=1, \ldots, p). \qquad (17)$$

From (16)

$$\sum_{k=1}^{t} b_{ij}^{(k)}(A_k X)=\sum_{k=1}^{t} b_{ij}^{(k)}(XM_k)$$

$$=X\sum_{k=1}^{t} b_{ij}^{(k)} M_k \qquad (i,j=1, \ldots, p). \qquad (18)$$

So if we let

$$D_{ij}=\sum_{k=1}^{t} b_{ij}^{(k)} M_k, \qquad (i,j=1, \ldots, p) \qquad (19)$$

then $D=(D_{ij})$ is the matrix defined by (15) and we have, from (17), (18), and (19)

$$C_{ij}X=XD_{ij} \qquad (i,j=1, \ldots, p), \qquad (20)$$

so that the matrices C and D satisfy (3). Thus, by part 2 of theorem 2, there exists a matrix P such that

$$P^{-1}CP=\begin{pmatrix} D & * \\ 0 & E \end{pmatrix}, \qquad (21)$$

so that pr of the roots of C are roots of D. Moreover, if the matrix X is known, D and E can be found explicitly by using (11) to construct P.

7. An Application to a Lattice Point Problem

The example below is a matrix which arose in a problem involving lattice points at the National Bureau of Standards:

$$\mathscr{A}=\begin{pmatrix}
4 & -1 & . & -1 & \beta & . & . & . & . \\
-1 & 4 & -1 & -\beta & -1 & \beta & . & . & . \\
. & -1 & 4 & . & -\beta & -1 & . & . & . \\
-1 & -\beta & . & 4 & -1 & . & -1 & \beta & . \\
\beta & -1 & -\beta & -1 & 4 & -1 & -\beta & -1 & \beta \\
. & \beta & -1 & . & -1 & 4 & . & -\beta & -1 \\
. & . & . & -1 & -\beta & . & 4 & -1 & . \\
. & . & . & \beta & -1 & -\beta & -1 & 4 & -1 \\
. & . & . & . & \beta & -1 & . & -1 & 4
\end{pmatrix}.$$

If we partition $\mathscr{A}$ into 3×3 submatrices we see

$$\mathscr{A}=\begin{pmatrix} A & B & 0 \\ B^T & A & B \\ 0 & B^T & A \end{pmatrix}. \qquad (22)$$

D. E. Rutherford [9] shows that a tridiagonal matrix,

$$T=\begin{pmatrix}
a & b & & & & & & & \\
b & a & b & & & & & & \\
 & b & a & b & & & & & \\
 & & & \cdot & & & & & \\
 & & & & \cdot & & & & \\
 & & & & & \cdot & & & \\
 & & & & & & b & a & b \\
 & & & & & & & b & a
\end{pmatrix}$$

has characteristic roots which may be written in the form $\lambda_k=a+2b\cos\theta_k$ where $\theta_k=k\pi/n+1$, $k=1, \ldots, n$.

The characteristic vector corresponding to λ_k is

$$x_k=(\sin\theta_k, \sin 2\theta_k, \ldots, \sin n\theta_k). \qquad (23)$$

The transformation for an i.r.p. tridiagonal matrix was given by the author in [6] and Rutherford also gives all the roots in the d.r.p. case.

The example (22) falls into none of the above categories, however, since $B \neq B^T$; but we show how the transformations discussed in section 4 can be used to simplify the matrix so that five of the roots may be found without having to solve an equation of degree higher than 2. I am grateful to D. E. Rutherford of the University of St. Andrews for giving me the other four roots.

We first use the transformation matrix, $P_1 = (X \times I)$ where X is the matrix of vectors (23) for a 3×3 tridiagonal matrix; i.e.,

$$X = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{2} & 1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ 1 & -\sqrt{2} & 1 \end{pmatrix}, \quad X^{-1} = X,$$

and I is the identity matrix of order 3. Then we find

$$\mathscr{B} = P_1^{-1}\mathscr{A}P_1 = \begin{pmatrix} A - \sqrt{2}\,I & C^T & 0 \\ C & A & C^T \\ D & C & A + \sqrt{2}\,I \end{pmatrix},$$

where

$$C = \frac{B - B^T}{2} = \begin{pmatrix} 0 & \beta & 0 \\ -\beta & 0 & \beta \\ 0 & -\beta & 0 \end{pmatrix}.$$

Since A and also the matrices $A \pm \sqrt{2}\,I$ are all tridiagonal, we now use the transformation matrix $P_2 = (I \times X)$ and find

$$\mathscr{C} = P_2^{-1}BP_2 = \begin{pmatrix} D - \sqrt{2}\,I & C^T & 0 \\ C & D & C^T \\ 0 & C & D + \sqrt{2}\,I \end{pmatrix},$$

where

$$\mathscr{D} = \text{diag}\,(4 - \sqrt{2},\ 4,\ 4 + \sqrt{2}).$$

If we write the rows and columns of $\mathscr{C}$ in the order, 826435791, we obtain the matrix

$$\mathrm{D} = \begin{pmatrix} E & 0 \\ * & F \end{pmatrix},$$

with

$$E = \begin{pmatrix} G & H \\ H & G \end{pmatrix}, \qquad F = \begin{pmatrix} K & L \\ L^T & M \end{pmatrix},$$

where

$$G = \begin{pmatrix} 4 + \sqrt{2} & \beta \\ \beta & 4 - \sqrt{2} \end{pmatrix}, \quad H = -I_2,$$

$$K = \begin{pmatrix} 4 & -\beta & . \\ -\beta & 4 & -\beta \\ . & -\beta & 4 \end{pmatrix}, \quad L = \begin{pmatrix} 0 & 0 \\ \beta & \beta \\ 0 & 0 \end{pmatrix},$$

and $M = \text{diag}(4 + 2\sqrt{2},\ 4 - 2\sqrt{2})$. Thus E is an o.r.p. circulant, so the roots of E are those of $G + H$ and $G - H$, which are $4 \pm \beta \pm \sqrt{\beta^2 + 2}$.

Since K is tridiagonal, let

$$P_3 = \begin{pmatrix} X & \\ & I_2 \end{pmatrix}.$$

Then

$$P_3^{-1}F_3 = \begin{pmatrix} 4 - \sqrt{2}\beta & . & . & \beta/\sqrt{2} & \beta/\sqrt{2} \\ . & 4 & . & . & . \\ . & . & 4 + \sqrt{2}\beta & -\beta/\sqrt{2} & -\beta/\sqrt{2} \\ \beta/\sqrt{2} & . & -\beta/\sqrt{2} & 4 + 2\sqrt{2} & . \\ \beta/\sqrt{2} & . & -\beta/\sqrt{2} & . & 4 - 2\sqrt{2} \end{pmatrix}.$$

Thus 4 is a root of this matrix, and when we delete the second row and column, we are left with a 4×4 matrix. I was unable to reduce this further by any relatively simple similarity transformation. However, D. E. Rutherford has found the other four roots of F:

$$4 \pm \sqrt{\beta^2 + 2\beta + 2} \pm \sqrt{\beta^2 - 2\beta + 2}.$$

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Bound for the P-Condition Number of Matrices With Positive Roots

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Let a matrix A have positive roots $0<\lambda_1\leq\lambda_2\leq\ \ldots\ \leq\lambda_n$. Upper and lower bounds for the P-condition number of A, $P=\lambda_n/\lambda_1$, are given in terms of det A and one other symmetric function of the roots.

1. Introduction

Suppose $A=(a_{ij})$ is a nonsingular $n\times n$ matrix with roots, $\lambda_i(i=1,\ \ldots\ ,n)$, ordered so that

$$0<|\lambda_1|\leq|\lambda_2|\leq\ldots\leq|\lambda_n|.$$

For a wide class of matrices A, the ratio $|\lambda_n|/|\lambda_1|$ gives a rough measure of the probable accuracy of the computation of the inverse of A, or the solution of the system of equations for which A is the matrix of coefficients.

This measure was evaluated in some detail by von Neumann and Goldstine [5][1] and has been called by J. Todd [2, 3, 4] the P- condition number of the matrix, i.e.,

$$P=\frac{|\lambda_n|}{|\lambda_1|}.$$

In general the accuracy of the results is in proportion to the reciprocal of P.

In this paper we shall show that, if the roots of A are all positive with

$$0<\lambda_1\leq\lambda_2\leq\ldots\leq\lambda_n \tag{1}$$

and the determinant of A, det A, is known, together with one other symmetric function of the roots, upper and lower bounds for P can be given in terms of these two known quantities.

More precisely, if the characteristic polynomial for the matrix A is

$$p(x)=x^n-c_1x^{n-1}+c_2x^{n-2}$$

$$-\ldots+(-1)^nc_n,\ c_i\geq 0 \qquad (i=1,\ \ldots,\ n) \tag{2}$$

and we know c_k and $c_n=(\det A)$, there exist positive upper and lower bounds for the ratio

$$P=\frac{\lambda_n}{\lambda_1}, \tag{3}$$

in terms of c_k and c_n.

To simplify the notation in the statement and proof of the theorem we use the following device.

[1] Figures in brackets indicate the literature references at the end of this paper.

Let

$$c_k=c_k(\lambda_1,\ \ldots,\ \lambda_n)=\binom{n}{k}s_k(\lambda_1\ \ldots,\ \lambda_n)=\binom{n}{k}s_k; \tag{4}$$

and divide each root of $p(x)$ by $(s_k)^{1/k}$. (This is equivalent to dividing each element of the matrix by $(s_k)^{1/k}$.) We will call the new matrix "normalized with respect to k" or simply the normalized matrix. Since the condition number, P, is the ratio of two roots, P will not be affected by such a transformation. Let D_k be the determinant of the normalized matrix. Then

$$D_k=\frac{s_n}{(s_k)^{n/k}}=\frac{\binom{n}{k}^{n/k}c_n}{c_k^{n/k}}. \tag{5}$$

From Hardy-Littlewood-Pólya [1] the numbers s_k as defined in (4) satisfy the inequalities

$$s_1\geq s_2^{1/2}\geq s_3^{1/3}\geq\ldots\geq s_n^{1/n} \tag{6}$$

(where equality holds only if all the λ_i are equal) hence, by (5), $D_k\leq 1$.

2. Statement and Proof of Theorem

THEOREM: *If we have a matrix $A=(a_{ij})$ with characteristic polynomial (2) and if the constant term and the kth coefficient are known, the following bounds hold for P,*

$$\frac{1}{D_k^{1/(n-1)}}\leq P\leq\frac{1+\sqrt{1-D_k^{n-1}}}{1-\sqrt{1-D_k^{n-1}}}, \qquad (k=1,\ \ldots,\ n-1) \tag{7}$$

where D is defined by (4) and (5).

If $k=1$ (i.e., the trace of the matrix is given) we can improve the upper bound:

$$P\leq\frac{1+\sqrt{1-D_1}}{1-\sqrt{1-D_1}}; \qquad D_1=\frac{n^nc_n}{c_1^n}. \tag{8}$$

PROOF: *Part 1—Upper bound.* We prove (8) first, as the method used for this can be applied in the proof of (7).

Since $P=\lambda_n/\lambda_1$, eq (8) is equivalent to

$$D_1\leq\frac{4\lambda_1\lambda_n}{(\lambda_1+\lambda_n)^2} \tag{9}$$

so from (5), we must show that

$$\frac{(\lambda_1+\lambda_n)^2}{4\lambda_1\lambda_n}\leq\left(\frac{s_1}{n\sqrt{s_n}}\right)^n. \tag{10}$$

But the right side of (10) is $(A_n/G_n)^n$ where A_n is the arithmetic mean, and G_n the geometric mean, of $\lambda_1, \ldots, \lambda_n$.

Thus, we can prove (10) by induction if we prove the following:

LEMMA:

$$\left(\frac{A_n}{G_n}\right)^n\leq\left(\frac{A_{n+1}}{G_{n+1}}\right)^{n+1}, \tag{11}$$

where A_{n+1} and G_{n+1} are the arithmetic and geometric means of $\lambda_1, \ldots, \lambda_n, \lambda_{n+1}$. (The λ_i are not assumed to be ordered with respect to size for this lemma.)

PROOF OF LEMMA: Simplifying (11) we see we must prove

$$\lambda_{n+1}A_n^n\leq A_{n+1}^{n+1}. \tag{12}$$

But (12) follows from the arithmetic-geometric mean inequality (cf., Hardy, Littlewood, Pólya [1]),

$$a_1a_2\ldots a_{n+1}\leq\left(\frac{a_1+a_2+\ldots+a_{n+1}}{n+1}\right)^{n+1},$$

if we let $a_i=A_n$, $(i=1, \ldots, n)$ and $a_{n+1}=\lambda_{n+1}$.

Thus (11) holds and (10) follows immediately using finite induction on n.

To prove (7) we note that, by the inequalities (6),

$$D_k^{n-1}\leq D_{n-1}^{n-1}=\frac{s_n^{n-1}}{s_{n-1}^n},$$

so it suffices to show that

$$\frac{(\lambda_1+\lambda_n)^2}{4\lambda_1\lambda_n}\leq\frac{1}{D_{n-1}^{n-1}}=\left(\frac{s_{n-1}}{s_n}\right)^n\cdot s_n. \tag{13}$$

Now let $\mu_1=\lambda_n^{-1}, \ldots, \mu_n=\lambda_1^{-1}$ and we note that $0<\mu_1\leq\ldots\leq\mu_n$ and (13) becomes

$$\frac{(\mu_1+\mu_n)^2}{4\mu_1\mu_n}\leq\left(\frac{s_1(\mu_1\ldots\mu_n)}{s_n^{1/n}(\mu_1\ldots\mu_n)}\right)^n. \tag{14}$$

But (14) is the same as (10) with the μ's substituted for the λ's, hence (13) holds and the general upper bound (7) is true for all values of k.

PROOF: *Part 2—Lower Bound.* From the inequalities (6),

$$D_k=\frac{s_n}{s_k^{n/k}}\geq\frac{s_n}{s_1^n},$$

where, by the definition (4)

$$s_1\leq\lambda_n, s_n\geq\lambda_1^{n-1}\lambda_n.$$

Hence

$$\frac{1}{D_k}\leq\frac{s_1^n}{s_n}\leq\frac{\lambda_n^n}{\lambda_1^{n-1}\lambda_n}=\left(\frac{\lambda_n}{\lambda_1}\right)^{n-1}$$

which proves the lower bound in (7).

3. Some Remarks

In reference [6] T. Kato gives an upper bound for the P-condition number which in our notation is

$$P<\frac{4}{D_1}.$$

This is larger than the bound given in (8).

We derive several consequences from (7), (8). Suppose that we have a matrix with positive roots, which is normalized so that $c_k=\binom{n}{k}$, i.e., $s_k=1$. Then $D_k=\det A$, and we can write

$$\frac{1}{(\det A)^{1/(n-1)}}\leq P\leq\frac{1+\sqrt{1-(\det A)^{n-1}}}{1-\sqrt{1-(\det A)^{n-1}}}. \tag{15}$$

This implies that

$$\det A\to 0 \quad \text{if and only if } P\to\infty,$$

$$\det A\to 1 \quad \text{if and only if } P\to 1.$$

Hence, the det A behaves essentially as the reciprocal of the P-condition number and may be said to constitute a reasonable condition number of its own for such matrices. This lends substance to the popular feeling that for a properly defined class of normalized matrices the smallness of its determinant is accompanied by difficulty in inversion.

A second observation relates to least square approximations. Express the general problem in the language of inner product spaces. We are required to solve $||y-\Sigma_{i=1}^n a_ix_i||=$minimum, where y is given and x_i are independent elements. The normal equations have matrix $((x_i,x_j))$ where (p,q) designates the inner product. We can assume x_1 normalized: $(x_i,x_i)=\|x_i\|^2=1$. The Gram matrix $((x_i,x_j))$ is positive definite symmetric and hence falls within the scope of our inequality with $k=1$, $c_1=n$. Hence if G is the Gram determinant $|(x_i,x_j)|$ we have

$$\frac{1}{G^{1/(n-1)}}\leq P\leq\frac{1+\sqrt{1-G}}{1-\sqrt{1-G}}. \tag{16}$$

The quantity G, which acts as a "measure" of linear independence of $x_1, \ldots, x_n$, therefore also serves as a condition number for the normal equations.

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(Paper 65B1–42)

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Some Computational Problems Involving Integral Matrices[1]

Olga Taussky[2]

(November 28, 1960)

In this mainly expository article dealing particularly with recent problems, a computational problem of G. Pall related to finite projective geometries is discussed in greater detail. Numerical results obtained on SEAC and SWAC are discussed.

1. Introduction

All computation on a machine with a fixed number of digits is arithmetic modulo some integer and therefore can be regarded as number theory. From this fact, in "analytic" problems there arise all the difficulties associated with "round-off," i.e., neglect of insignificant digits. However, there are often considerable difficulties in handling actual number theoretical problems, e.g., because the numbers may grow large rapidly. Codes which permit numbers of arbitrary length have been written, but they have to be monitored carefully.

Matrices with integral elements have been studied for a very long time and an enormous number of problems arise, both theoretical and practical. The whole vast classical theory of quadratic forms comes under the theory of symmetric matrices; many new problems have arisen in recent years. Some of the problems can be divided, perhaps, into four topics:

(1) Inversion (see e.g., J. B. Rosser [1]).

(2) Eigenvalues and eigenvectors. These play a role in the determination of the class number of an algebraic number field F by rational methods. For it can be shown that the number of ideal classes coincides with the number of classes of matrices $S^{-1}AS$, where A is a matrix with integral elements which is a root of the algebraic equation determining F, and S runs through all unimodular matrices with integral elements (see O. Taussky [2, 3]).

(3) Enumeration of matrices with special properties. Here we have the various problems concerning latin squares, the problems concerning matrices with ± 1 as elements and orthogonal rows (see e.g., R.E.A.C. Paley [4] and J. Hadamard [5]). The study of finite projective geometries has led to new problems in this field and some of them will be discussed here later. A generalization of these problems is given by the study of block designs which are of importance in statistical work (see M. Hall [6], H. Ryser [7], R. C. Bose [8], W. S. Connor and W. H. Clatworthy [9]).

(4) Maximization problems. These come actually also under the problems discussed in (3), but have been studied rather extensively, particularly in recent years. A prototype among these is the assignment problem: given an $n \times n$ matrix (a_{ij}), determine the permutation $P(i)$ for which

$$\sum a_{iP(i)}$$

is maximum (see e.g., T. S. Motzkin [11], C. B. Tompkins [12], H. W. Kuhn and A. W. Tucker [13]).

2. Some Problems

However, many other problems arise which cannot easily be brought under these headings. Of these we mention a few. Matrices with rational integral coefficients are a generalization of rational integers and any number theoretical process can be applied to them: e.g., any two $n \times n$ matrices of this type have a greatest common left divisor, a greatest common right divisor, a least common left multiple, a least common right multiple. If D is the greatest common left divisor of A,B, then

$$D = AP + BQ$$

can be solved where P, Q are again $n \times n$ matrices with rational integral coefficients. Diophantine equations can be studied, even the Fermat equation

$$x^n + y^n = z^n$$

can be considered for matrices.

Completely new problems arise as well because of the noncommutativity of matrices. A quite elementary example is to express a matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, with a, b, c, d rational integers and $ad - bc = 1$ in terms of the generators $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. A much more difficult problem is to express (when possible) a positive definite $n \times n$ matrix C of rational integers in the form $C = XX'$ where X is again an $n \times n$ matrix of rational integers and X' is its transpose. If the determinant $|C| = 1$ and $n < 8$ such a decomposition is known to be possible always. Actually to find X can be a difficult computational problem (see [7]).

[1] This paper is based on an invited address to the Società Italiana per il Progresso delle Scienze, in Sicily in 1956; it was prepared with partial support from the Office of Naval Research.

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3. Hermite Normal Forms

For many of the problems concerning matrices of rational integers the so-called Hermite normal form of a matrix A plays an important role. It is a triangular matrix obtained by multiplying A with a suitable unimodular matrix U of integers and is of the form.

$$UA=\begin{pmatrix} h_{11} & h_{12} & . & . & h_{1n} \\ 0 & h_{22} & . & . & h_{2n} \\ . & . & . & . & . \\ 0 & 0 & . & 0 & h_{nn} \end{pmatrix},$$

where $h_{ii}\geq 0$ and $0\leq h_{ik}<h_{ii}(i<k)$.

Another important normal form associated with A is the Smith normal form which is a diagonal matrix. It is obtained by multiplying A with two unimodular matrices U, V so that

$$UAV=\begin{pmatrix} h_1 & & & & & & \\ & h_2 & & & & & \\ & & . & & & & \\ & & & . & & & \\ & & & & h_\rho & & \\ & & & & & 0 & \\ & & & & & & . \\ & & & & & & & . \\ & & & & & & & & 0 \end{pmatrix}$$

and $h_1|h_2|\ .\ .\ .\ |h_\rho$.

4. Application of the Hermite Normal Form to Finite Projective Planes

An important use of the Hermite normal form was made in the study of finite projective plane geometries (FPP). An FPP is a set of k elements called points which are distributed over k lines such that exactly $n+1$ points lie on each line and exactly $n+1$ lines pass through a given point. Two distinct points are contained in one and only one line; two distinct lines contain one and only one point in common. We have $k=n^2+n+1$. The number n is called the order of the geometry. With such a geometry there is associated the so-called $k\times k$ incidence matrix

$$A=(a_{ij})$$

where

$a_{ij}=1$ if the ith point lies on the jth line,
$a_{ij}=0$ if the ith point does not lie on the jth line.

Hence any two rows (columns) have a one in common in exactly one column (row) and there are $n+1$ elements $a_{ij}=1$ in each row (column).

The incidence matrix A is a matrix of integers. If the lines (points) are numbered in a different order then A is replaced by PA (AQ) where P and Q are permutation matrices. The following two properties of A are particularly important:

$$\det A=\pm n^{(n^2+n)/2}(n+1),$$
$$AA'=A'A=nI+J,$$

where I is the unit matrix and J the matrix consisting of 1's only.

G. Pall studied the Smith and Hermite normal forms of A (see [14]). He showed that for square free values of n the diagonal elements in the Smith form are

$$1, 1, \ .\ .\ .\ , 1, n, \ .\ .\ .\ , n, n(n+1)$$

and the diagonal elements of the Hermite normal form are 1, or divisors of n, or $(n+1)$ times a divisor of n.

Two different geometries cannot have the same Hermite matrix. Thus the Hermite matrix of a geometry is rather important. G. Pall further investigated the p-rank of A, i.e., the maximum number of linearly independent rows of A, mod p, where p is a prime factor of n. If p^2 is not a factor of n (e.g., if n is square free) then the p-rank of A is exactly

$$\frac{1}{2}(n^2+n)+1.$$

For all other p

$$p\text{-rank}\leq\frac{1}{2}(n^2+n)+1$$

holds. As special examples the 2-rank for the (unique) geometry corresponding to $n=4$ was computed. It is 10. The 3-rank corresponding to $n=9$ presented a more difficult task. There are two different geometries of order 9. The 91×91 incidence matrices of these geometries have as 3-rank 37 and 41 while $\frac{1}{2}(n^2+n)+1=46$. These values of the 3-rank were computed on SEAC and on SWAC. The computation on SEAC was carried out by M. Newman; it took 20 hr. The Hermite normal forms of the geometries of order $n=5$ and $n=7$ were also computed; it took 3 hr for this task.

5. Further Facts About the Incidence Matrix[3]

Incidence matrices have been used to study the still unsolved problem: For what value of n is there a finite projective plane? So far it is known that for $n=p^r$, p a prime number, there is always a

[3] See also reference [15].

finite projective plane. This plane is uniquely determined for $n=2, 3, 4, 5, 7, 8$. This was established for $n=7$ by M. Hall [16] and for $n=8$ by computations carried out on SWAC by M. Hall, J. D. Swift and R. J. Walker [17].

For $n=6$ there is no FPP whatsoever (see R. H. Bruck and H. J. Ryser [18], S. Chowla and H. J. Ryser [19]). On the other hand for $n=9$ it is known that there exist two different geometries; one of these satisfies the axiom of Desargues, the other does not. For $n=10$ it is not known whether a FPP exists.

A valuable contribution towards the solution of the problem of determining the possible orders of a FPP is given by the following theorem of Ryser [20]:

Let n *be odd and let* $k=n^2+n+1$. *Assume* $nI+J=XX'$ *where* X *is a matrix of integers. Then there exists a* FPP *with* $n+1$ *points on each line.*

However, as pointed out earlier, it is a rather difficult computational problem of deciding whether a given positive definite matrix of high order splits up in the form XX' with X an integral matrix.

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Computational Problems Concerning the Hilbert Matrix[1]

John Todd [2]

(November 21, 1960)

The interaction between theoretical mathematics and practical computational expiriment is illustrated by a discussion of recent work, at various centers, concerning the Hilbert matrix.

1. Introduction

Recent work, in various centers, concerning the Hilbert matrix shows very well the interaction between (theoretical) mathematics and practical computational experiments. Some aspects of this work will be discussed here in an expository manner. Although we shall be concerned mainly with the finite segments H_n of the (infinite) Hilbert matrix

$$H=\{h_{i,j}\}=\{(i+j+1)^{-1}\}, \qquad i,j=0,1,2,\ldots,$$

many of the results can be generalized, e.g., to matrices of the form

$$H=\{(a_i+b_j)^{-1}\}, \qquad i,j=0,1,2,\ldots,;$$

or

$$H(\theta)=\{(i+j+\theta)^{-1}\}, \qquad i,j=0,1,2,\ldots, \quad 0<\theta<1.$$

2. Theory

The literature of the last half-century contains many theoretical results concerning the Hilbert matrix. Of these only the basic inequality is immediately relevant [8, p. 234]:

2.1. *If* $\sum_0^\infty a_m^2$ *is convergent, then*

$$\sum_0^\infty \sum_0^\infty a_m a_n/(m+n+1)<\pi \sum_0^\infty a_m^2.$$

We note the following consequence of this.

2.2. *The equation* $Hx=\pi x$ *cannot be satisfied by any vector* x *in Hilbert space,* l^2.

Suppose

$$Hx=\pi x$$

is satisfied for a vector $x \epsilon l^2$. Then it is permissible to multiply both sides by x', i.e., take inner products, to get

$$x'Hx=\pi x'x.$$

If $x=(x_0,x_1,\ldots,\ldots)$ so that Σx_m^2 is convergent this gives

$$\Sigma\Sigma x_m x_n/(m+n+1)=\pi\Sigma x_m^2,$$

in contradiction with (2.1).

The question as to whether there is an x, necessarily not in l^2, for which the equation

$$Hx=\pi x$$

is true, was raised by O. Taussky [20]. This question will be discussed in sections 5 and 6 below.

3. Applications

Among the areas in applied mathematics in which the Hilbert matrix or related matrices have turned up are aerodynamics (A. R. Collar [3]), diffraction of electromagnetic waves (W. Magnus and F. Oberhettinger [12]), and statistics (I. R. Savage and E. Lukacs [17]).

4. Computational Problems—Inversion

Theoretically, the problem of inverting the finite matrices H_n is solved because of the existence of an explicit representation of the inverse, a result which is, essentially, due to Cauchy. The matrix is of the form known as a double-alternant, and the inversion has been discussed e.g., by A. C. Aitken [1, p. 134], A. R. Collar [3], and G. Pòlya and G. Szegö [15, pp. 98–99, 299–300]. The explicit form of the inverse, and the actual numerical values for $n=1(1)10$, have been given by I. R. Savage and E. Lukacs [17].

Practically, however, the problem remains interesting. For instance, although the Hilbert matrix itself turns up in idealized situations, it may be expected that slightly different matrices turn up in the practical circumstances which are approximated by these. It is, therefore, desirable to study the mechanical inversion of the Hilbert matrices H_n, especially as they enjoyed a reputation for ill-condition i.e., "numerical instability," long before the advent of high-speed computers and the basic papers of von Neumann and Goldstine [25] and Turing [24], in order to compare the observed inverse with the theoretical one.

The Hilbert matrix and related ones (e.g., Lotkin [10]) have been often used to test matrix inversion

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[1] This paper is based on an invited address to the Società Italiana per il Progresso delle Scienze, in Sicily in 1956; it was prepared with partial support from the Office of Naval Research.

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programs prepared for high-speed computers. All the experiments of which we are aware [2, 16, 23]—and in these several different schemes for inversion were used—completely confirm the bad reputation. The results quoted by Todd [23] are fully representative. For machines with 10 to 12 decimals, with floating point arithmetic to about 8 significant figures, there is rapid deterioration of the quality of the alleged inverses which are produced—and usually complete failure to produce any inverse for H_n with n about 8.

A detailed examination of the condition of H_n has been given by Todd [23]. Given a matrix A and a specific process of inversion, the error in computing A^{-1} can be found, in theory. In practice what is wanted is a cheap estimate of this, in terms of quantities associated with the matrix and e.g., the word-length of the machine. Various condition-numbers have been introduced to measure the condition of the matrix. One of these, the P-condition number is defined as $P(A)=|\lambda|/|\mu|$ where $|\lambda|$ and $|\mu|$ are the greatest and least among the absolute values of the characteristic values, of A. It has been shown [23], that α being a certain positive constant,

$$P(H_n)=O(e^{\alpha n})$$

while for the $n\times n$ matrix

$$C_n=\begin{pmatrix} -2 & 1 & & & & \\ 1 & -2 & 1 & & & \\ & 1 & -2 & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & 1 & -2 \end{pmatrix}$$

associated with a second-order differential equation, we have

$$P(C_n)=O(n^2).$$

Using these results in the general error-estimate obtained by von Neumann and Goldstine [25], for an elimination process, and comparing these theoretical error-estimates with actual observed estimates, shows that the two are in reasonable agreement (see Todd [22]).

5. Computation Problems—Characteristic Roots and Vectors

As soon as programs for the computation of characteristic roots and vectors were constructed, it was natural to test them on the H_n, since input programs for this had already been constructed. There was no reason to suppose that it would be particularly difficult to handle—since it is considered that the presence of multiple roots, or roots close in absolute value would be the most likely source of trouble. The theory of positive matrices due to Perron and Frobenius (see e.g., Wielandt [26]) guarantees the existence of a single dominant characteristic root with a characteristic vector all components of which are positive, so that the power method should be satisfactory. This is indeed the case and the dominant root, and the corresponding vector have been obtained for values of n up to 200 at various centers [5, 18].

If more characteristic roots and vectors are needed, the Gantmakher and Krein [6] theory of completely positive matrices guarantees that the characteristic roots are all simple and positive and the components of the vectors exhibit a characteristic pattern: the first has all its components positive and each succeeding one has one more change of sign.

The last criteria shows up the weakness of the results obtained. For instance, a reasonably efficient code using floating point with 8 significant figures which gives the dominant root λ_5 of H_5 correct to 8S gives the smallest root negative! Actually,

$$\det H_5 \doteq 3.75\times 10^{-12}$$

and

$$5.58\times 10^{-6}\geq \mu_5 \geq 1.12\times 10^{-6}$$

so the results are not too surprising.

The usual method for handling this case is an approximate diagonalization of the matrix using the Jacobi process, or the variation on this due to W. Givens [7] in which the matrix is reduced to triple diagonal form and the roots located by a Sturmian process. So far few actual results of the use of these methods have been published.

6. More Theory

Following experimental computations, O. Taussky [19] showed that, if λ_n denotes the dominant characteristic root of H_n, then

$$\lambda_n=\pi+O(1/\log n)$$

so that $\lambda_n\to\pi$. The actual approach to π is very slow, e.g., it is known [5,18] that: $\lambda_{50}\doteq 2.08$, $\lambda_{60}\doteq 2.11$, $\lambda_{75}\doteq 2.14$, $\lambda_{100}\doteq 2.18$, $\lambda_{125}\doteq 2.21$, $\lambda_{200}\doteq 2.27$; the exact order of magnitude of $\pi-\lambda_n$ does not appear to be known.

A constructive attack on the problem, already mentioned in section 2, of the existence of a vector v satisfying

$$Hv=\pi v$$

appeared difficult and an affirmative solution to it was obtained by T. Kato [9], who used an indirect approach. His method, which will now be described, was suggested by observations of the monotonic behavior of the coordinates of the dominant characteristic vectors of H_n, when these are normalized so that the first coordinate is unity.

Suppose the characteristic vector v_n of H_n corresponding to λ_n is

$$v_n=(v_1^n, v_2^n, \ldots, v_n^n), \qquad v_1^n=1.$$

Numerical evidence suggests that both the sequence $\{\lambda_n\}$ and all the sequences $v_i^t, v_i^{t+1}, v_i^{t+2}, \ldots$ $(i=1,2, \ldots)$ are monotone. The first fact is known (e.g., Collatz [4]) and the second is a special case of a more general result regarding the dominant vectors a, b of two positive matrices A, B where A is $\alpha \times \alpha$ and B is $\beta \times \beta$. If B dominates A in the sense that $\alpha \leq \beta$ and if $c_{ij}=b_{ij}/a_{ij}$ is a nondecreasing function of i, j (as long as it is significant), then b dominates a in a similar sense, provided further that B has all its second-order minors positive. This is proved using the fact (already noted) that in these circumstances the dominant vectors can be obtained by the "power" method. The special case used is that in which $A=H_n$, $B=H_{n+1}$.

Now assume that λ_n is bounded—that this is the case when we are dealing with H_n, follows from (2.1). Then $\lim \lambda_n=\lambda$, where $\lambda=\pi$ in the Hilbert case. It follows that the sequences $v_i^t, v_i^{t+1}, v_i^{t+2}, \ldots$ are bounded, for comparing the first coordinates in the equation

$$H_n v_\eta=\lambda_n v_n$$

we have

$$h_{0,i}\, v_i^n \leq \sum h_{0,j}\, v_j^n=\lambda_n\, v_1^n=\lambda_n \leq \lambda,$$

which gives, for all n,

$$v_i^n \leq \lambda (h_{0,i})^{-1}.$$

In the Hilbert case this is

$$v_i^n \leq \pi(i+1).$$

If we write $v_i=\lim_n v_i^n$, it can then be shown that the infinite vector $v=(v_1, v_2, \ldots)$ satisfies

$$Hv=\pi v.$$

The result obtained by Kato [9], while answering the original question, itself suggests many more. For instance, is the vector v the only (linearly independent) vector corresponding to $\lambda=\pi$? Are these characteristic values which exceed π?

7. Recent Developments

1. During the last few years the theory described in section 6 has been developed considerably by Tosio Kato and Marvin Rosenblum, and some of the questions raised there have been answered. Among the relevant results are the following:

M. Rosenblum [28] has shown that every complex number with a positive real part is a characteristic root of $H(\theta)$ for θ fixed, $\theta>0$. This was accomplished by using the theory of special functions to obtain a characteristic vector explicitly. In [29] Rosenblum completely determined the spectrum of $H(\theta)$, for any real $\theta \neq 0, -1, -2, \ldots$; this was accomplished using the Titchmarsh-Kodaira theory of singular differential operators.

T. Kato [30] shows that if $M(\theta)$ is the Hilbert bound of $H(\theta)$, i.e. $M(\theta)=\pi$ cosec $\pi\theta$, $0<\theta \leq \frac{1}{2}$; $M(\theta)=\pi$, $\theta \geq \frac{1}{2}$, then every $\lambda \geq M(\theta)$ is a characteristic value of $H(\theta)$, with a positive characteristic vector and there is no characteristic value $\lambda<M(\theta)$ which has a positive characteristic vector.

2. A reference to Hilbert's own work about H_n is [27] where he evaluates det H_n. For some related results see R. B. Smith [32], W. W. Sawyer [33]. The problem of matrix inversion is discussed from the experimental point of view in [31], which includes further results on the Hilbert matrix.

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Index to the Distributions of Mathematical Statistics

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A fairly complete index of references to results on statistical distributions published before January 1958 is presented. The material given for each distribution is a list of references relating to: (*a*) functions and constants which characterize the distribution, (*b*) derived distributions, (*c*) estimation, (*d*) testing statistical hypotheses, (*e*) miscellaneous. The distributions covered are characterized as normal, type III, binomial, discrete, distributions over (a,b), distributions over (a, ∞), distributions over $(-\infty, \infty)$, miscellaneous univariate, miscellaneous bivariate, and miscellaneous multivariate. The number of entries varies from one or two for less well-known distributions to several hundred for the normal distribution. This index should serve to eliminate unnecessary derivation of results already in the literature.

I. Introduction

1. Background

The author began this index in April 1954 with the limited intention of supplying his students at Auckland University College, New Zealand, with a small reference pamphlet. It appeared that no textbook of mathematical statistics contained a complete treatment of all the distributions that a student might encounter. Thus an index was needed to facilitate selection of the appropriate book. In 1955 the collected results were mimeographed, and during the next three years several hundred copies of this early version were sent out in response to requests.

Then, at the invitation of the National Bureau of Standards, the author spent the summer of 1958 at the Bureau's Statistical Engineering Laboratory supplementing and editing the index for publication in the present form.

In the course of this work, a number of additional sources have been included and the journal coverage has been extended through 1957. Information supplied by readers of the original version has been incorporated, and various mistakes have been corrected.

2. Organization

The material given under each distribution consists of a number of *entries*, most of which are provided with one or more *references*. In the case of the normal distribution with mean m and variance v (No. 1.1) the number of entries is fairly large, and therefore the standard order is most easily seen:

A. Functions and parameters
B. Derived distributions
  (*a*) of linear quantities
  (*b*) of quadratic quantities
C. Estimation
  (*a*) point
  (*b*) interval
D. Testing statistical hypotheses
  (*a*) by linear statistics
  (*b*) by quadratic statistics
E. Miscellaneous

These categories are by no means always used for less important distributions. With the limited information available, a complete listing of the headings in such cases would be wasteful as the majority would be empty. Keeping in mind the above order, it should not be difficult to find the required entry.

Occasionally an entry will be indented; such an entry should be read as a continuation of the preceding one.

3. References

The references to the literature are of the following types:

A. Coded.

(*a*) Journals, e.g., [c]4:17, which refers to page 17 of the 4th volume of the journal designated as [c] in table 1.

(*b*) Books, e.g., [12]53, which refers to page 53 of the book designated as [12] in table 2.

B. Uncoded, e.g., Trans. Am. Math. Soc. 17:382, conforming to the usual volume and page reference style.

C. Reviews

(*a*) *Mathematical Reviews* is designated by MR,

(*b*) *Zentralblatt für Mathematik* is designated by Z.

MR and Z references will in no case offer a review of a paper appearing in coded journals and therefore may be considered to indicate publications in obscure (from the point of view of the present work) sources. Moreover, every effort has been made to avoid MR or Z references to an uncoded paper quoted and very few duplications of this sort should be found.

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The choice between direct (i.e., coded or uncoded) and indirect (i.e., review) references is frequently available. The one that was actually inspected is given. All direct references were collected before the search of MR and Z. Consequently, each entry corresponding to a direct reference is based on the paper, and never its review, and each entry corresponding to an indirect reference is based on the review and never the paper.

As it is difficult to distinguish priority in a large number of references, a chronological table of the coded and review references is provided. This table also shows what volumes have been systematically searched in the preparation of this index.

4. Criteria for Inclusion

A. Distributions. As a general principle, a distribution is included if its density (or probability) function is a known, explicit function. The following exceptions may be noted:

(*a*) Certain families of distributions are mentioned, e.g., Pearson and Koopman, whose densities are specified only implicitly.

(*b*) Certain distributions are mentioned in terms of their cumulative probability function or characteristic function.

B. Entries. The general principle governing the selection of entries is that each entry must exhibit a property of the distribution in question. Exceptions to this rule generally take one of the following forms:

(*a*) Historical information about well-known distributions, although not systematically sought, may in some circumstances be included.

(*b*) Important applications, such as those which led to the discovery of the distribution, are usually supplied.

(*c*) Bibliographies.

Reference to tables has been excluded in almost every case.

It is clear that applications must be severely limited. With a slight exaggeration, several whole branches of statistics may be considered applications of some particular distribution, as exhibited for example in the following table:

Distribution	*Application*
Binomial	Quality control
Normal	Analysis of variance
Lognormal	Probit analysis
Poisson	Random processes
Deterministic	Applied mathematics

5. Relationship Between Distributions

In some cases (such as 2.1 and 2.3) the relationship between two distributions is asserted in their designation. In others (such as 5.3 and 5.15) a very close connection is not pointed out. In most cases, however, known relationships are simply listed among the miscellaneous properties of both distributions. In choosing between these procedures, an attempt has been made to follow current statistical usage and terminology.

Very similar principles have been used to decide for or against independent listing. If one distribution is relatively important and its equivalent much less so (for example Chi-square and Erlang), they are listed together. In other cases independent reputation seems to justify independent categories.

It must certainly be supposed that many of the trivial distributions of section 8 could be included in some larger category or even combined with each other. Such a systematic classification which would exhibit all connections, even if worth doing, is certainly removed from the purpose of this index, and has hardly been attempted.

For example, it is well known that no. 8.1 contains as special cases all the distributions of sections 1 and 2; very likely it also contains dozens of others listed. Nevertheless, to indicate this situation by a system of subheadings, applied to all entries, would quickly undermine the utility of the whole work, since it is the special case, rather than the general principle, that occurs in statistical practice.

6. Notation and Terminology

In univariate distributions the stochastic variable is always denoted by x, in bivariate by x and y and in multivariate by $x_1, \ldots, x_k$, quite regardless of the domain of definition. This usage departs from that of certain authors in two respects:

(*a*) The letter n is not used for a discrete variable.

(*b*) The statistic obeying a particular distribution is not used in the density. For example in Student's "t" distribution, we write

$$\left(1+\frac{x^2}{\nu}\right)^{-\frac{1}{2}(\nu+1)}$$

rather than

$$\left(1+\frac{t^2}{\nu}\right)^{-\frac{1}{2}(\nu+1)}$$

This practice is justified not only by the need for uniformity but by the belief that the alternative is wasteful of the alphabet: t, F, z, D, ... Similarly we prefer to call distributions by the names of their discoverers (or reputed discoverers) rather than by the symbol used to denote some statistic found to satisfy them. Of course, all known designations are found in the final index.

In many books the expression f(x) is employed to denote a probability density. However f is commonly used in mathematics for an arbitrary function. D(x) has therefore been selected as a more distinctive means of representing this special function.

In the discrete case D(x) replaces the probability distribution, which is often written p_n. C(x) is the cumulative function.

When we come to the characteristic function the situation is a little more complicated. Using t for

the variable, statistical works generally have to define several symbols for characteristic functions of various quantities, for example:

$\chi(t)$ = characteristic function of distribution of x
$\phi(t)$ = characteristic function of distribution of $n\bar{x}$
$\xi(t)$ = characteristic function of distribution of $\bar{x}$, etc.

As we will be dealing with many different statistics and possibly their characteristic functions, it is more economical and systematic simply to abbreviate by the following system: Ch(x), Ch $(n\bar{x})$, Ch $(\bar{x})$, etc. Thus it is not necessary to select a new letter to denote the characteristic function of each new statistic.

However, this practice leads to equations like

$$\mathrm{Ch}(x) = e^{-\frac{1}{2}vt},$$

which may be offensive to some, however clear the meaning. Such readers are advised to interpret the equality sign as an abbreviation for the verb "is."

This interpretation has another important connection with the notation being used. A variety of verbs has been employed to describe the relation between a stochastic variable and its distribution, for example:

x *obeys* the normal distribution with mean m and variance v,
x *follows* the normal distribution with mean m and variance v,
x *is* a normal variable with mean m and variance v.

It seems equally felicitous to assert this relationship by the convenient abbreviation

$$D(x) = N(m,v),$$

which may, if preferred, be regarded not as a mathematical equation but as shorthand. In any case it makes possible an unambiguous condensation of the facts.

Similar remarks apply to the expressions MGF(x), FD(p) which are used to mean moment generating function of the distribution of x and fiducial distribution of the parameter p.

Another application of this use of the equality sign relates to the symbols C.—R.(p), MLE(p), MME(p), UMVUE(p), BANE(p), and is exemplified by the following:

$$\text{C.—R.}(\sigma) = v/2n$$

$$\mathrm{MLE}(v) = s^2$$

$$\mathrm{UMVUE}(v) = ns^2/(n-1).$$

For the meanings of these and other abbreviations, the reader is referred to the following list:

D(x)—Density or probability function of a stochastic variable x

C(x)—Cumulative distribution function of x

Ch(x)—Characteristic function of distribution of x

MGF(x)—Moment generating function of distribution of x

PGF(x)—Probability generating function of x

FD(p)—Fiducial distribution of a parameter p

m—Mean of a population

$\bar{x}$—Mean of a sample

$v = \sigma^2$—Variance of a population

s^2—Variance of a sample

(i.e., $s^2 = \sum_{i=1}^{n}(x_i - \bar{x})^2/n$)

μ_k—kth central moment of a population

α_k—kth moment about the origin for a population

K_k—kth cumulant

r—Correlation coefficient in a sample

ρ—Correlation coefficient in a population

ξ—Median

GM—Geometric mean

HM—Harmonic mean

n—Number of items in a sample

β—Slope of regression line in a population

b—Slope of regression line in a sample

$\sim$—Asymptotic (=large sample)

β_1, β_2—Pearson's betas

C.—R.(p)—Cramér-Rao lower bound for variances of estimates of the parameter p

MLE(p)—Maximum likelihood estimate of the parameter p

MME(p)—Minimax estimate of the parameter p

Mχ^2E(p)—Minimum Chi-square estimate of the parameter p

UMVUE(p)—Uniformly minimum variance unbiased estimate of the parameter p

BANE(p)—Best asymptotically normal estimate of the parameter p

LR—Likelihood ratio

L—The likelihood function $\Pi\, D(x_i)$

Seq—Sequential

OC—Operating characteristic

BCR—Best critical region

Q—A quadratic form

E—Expectation

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Finally, it is too much to hope that a work of this character and magnitude is entirely free from error. The author will welcome notification of necessary corrections, and of important omissions.

II. Distributions

1. Normal

1.1. Normal (m, v)

A. *Functions and Parameters*

$D(x)=(1/\sqrt{2\pi v})e^{-(x-m)^2/2v}$: [6]108, [5]34, [4]57, [8]91, [9]243.

$Ch(x)=\exp(-\frac{1}{2}vt^2+mit)$: [1]211, [5]62.

$MGF(x)=\exp(\frac{1}{2}vt^2+tm)$: [6]112.

Derivatives etc. [d]2:181.

Transformations: [c]39:290.

Obtained from Pearson's differential equation: [4]72.

Called Type VII: [11]45.

Limit of binomial: [4]58.

Variance of $\bar{x}$ and s: [3]42.

$Var(m_3)=6v^3n^{-1}$, $var(m_4)=96v^4n^{-1}$ and many other constants: [2]224.

Calculation of constants and numerical examples: [11]88.

Mean deviation $E|x-m|=(2v/\pi)^{\frac{1}{2}}=.79788\sigma$: [1]258.

Probable error $=.6745\sigma$: [4]58.

$\alpha_{2k}=(2k-1)v^k$: [5]XII, [8]98.

Quasi-range: [d]24:603.

B. *Derived Distributions*

$D(\bar{x})=N(m, vn^{-1})$: [9]270, [6]10.2, [2]243, [4]100, [w]1:93.

$D(\bar{x}/s)$: [3]139.

$D[(n-1)^{\frac{1}{2}}s^{-1}(\bar{x}-m)]=$Student $(n-1)$: [6]217, [5]98, [2]239, [4]112, [w]1:74.

$D[s^{-1}(n-1)^{\frac{1}{2}}\bar{x}]$, where x_i are samples from $N(m_i,\sigma)$; m_i not all equal: [d]19:406.

$D[(\bar{x}-m)/\text{range}]$: [d]22:469.

D (range) etc.: MR13:762.

$D(\Sigma k_i x_i)=N(\Sigma k_i m_i, \Sigma k_i^2 v_i)$: [4]99, [8]92.

$[(x-m/\sigma)]^2$ is Chi-square, $(x/\sigma)^2$ is noncentral Chi-square, $(x-m)^2$ is Type III, product of normal variables is Bessel, quotient ~normal for large m/σ: [18]1–150.

$$D\left[\frac{m_1-m_2(x_1/x_2)}{\sqrt{v_1+v_2(x_1^2/x_2^2)}}\right]=N(0,1),\quad m_2\gg\sigma_2\text{: [2]253.}$$

$D(\bar{x}_1-\bar{x}_2)=N(m_1-m_2, v_1/n_1+v_2/n_2)$: [4]100.

$$D\left[\frac{\bar{x}_1-\bar{x}_2}{\sqrt{v\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}\right]=N(0,1)\text{ when }m_1=m_2\text{ and }v_1=v_2=v\text{: [6]263.}$$

$$D\left[\frac{(\bar{x}_1-\bar{x}_2)-(m_1-m_2)}{(n_1s_1^2+n_2s_2^2)^{\frac{1}{2}}}\left(\frac{n_1+n_2-2}{n_1^{-1}+n_2^{-1}}\right)^{\frac{1}{2}}\right]=\text{Student }(n_1+n_2-2)\text{: [4]112, [3]109, 112, [5]98, [c]29:350, [c]33:252, MR8:42.}$$

$$D\left[\left(\frac{n_1+n_2-2}{2}\right)\left(\frac{n_1(\bar{x}_1-m_1)^2+n_2(\bar{x}_2-m_2)^2}{n_1s_1^2+n_2s_2^2}\right)\right]=\text{Snedecor }(2, n-2)\text{, confidence ellipse: [4]132.}$$

$D(k^{th}$ value from top): [1]374, [d]25:565.

D(smallest sample value) = No. 8.40: [g]42:408.

$D[\frac{1}{2}v^{-1}(x-a)]=$Gamma $(\frac{1}{2})$: [10]150.

D(HM): MR4:164.

$D(\sqrt{x^2+y^2})$, $D(\sqrt{x^2+y^2+z^2})$ in special circumstances: MR16:377.

$D(\Sigma x_i^2)$: [1]3:353.

$$D(\chi^2)=D\left(\sum\frac{x_i-m_i}{\sigma_i}\right)^2=\chi^2(n)\text{: [6]10.3.}$$

$D(s^2)=$Type III $(n/2v, \frac{1}{2}n)$: [d]5:281, [6]10.3.

$D[(1/n)\Sigma(x_i-x)^2]=$Type III $[(\frac{1}{2}nv^{-1}, \frac{1}{2}(n-1)]$: [6]10.4.

D(s) for n = 2,3: [c]11:277.

$D(s/\bar{x})$, Coefficient of variation: [d]7:129, [a]94:564, [a]95:695.

D(range/s): [d]17:366, [c]31:20.

$D[(2s^2)^{-\frac{1}{2}}]$, "precision constant" = Type V, moments, etc.: [d]3:20, [a]97:132.

$D(\log s^2)$: [b]8:128.

$D(ns^2v^{-1})=\chi^2(n-1)$: [3]115

For unequal v,

$D[n_1(n_2-1)v_2s_1^2/n_2(n_1-1)v_1s_2^2]=$Snedecor$(n_1-1, n_2-1)$: [4]115, [10]197.

Testing and confidence intervals: [d]13:371.

Power function: [d]17:182.

For equal v

$D(n_1 s_1^2 v^{-1} + n_2 s_2^2 v^{-1}) = \chi^2(n_1+n_2-2)$,

$D\left(\frac{1}{2}\log\frac{n_1(n_2-1)s_1^2}{n_2(n_1-1)s_2^2}\right)$ = Fisher (n_1-1, n_2-1): [10]198.

$D(v^{-1}\Sigma\,(n_1-1)s_1^2) = \chi^2$: [4]116.

D(variance ratio): [k]11:136.

Distribution of various statistics from k normal populations with common variance: [e]17:2.

Ranking variances: [g]51:621.

Distribution of many quantities in a wide variety of cases: *J. Soc. Stat. Paris 96:262.*

D(various Q): *MR13:142.*

$D(\bar{x},s)$: [2]238, *MR8:161.*

$D(b_2)$ for $n=4$ is hypergeometric: [c]25:411.

$D(b_1)$ for $n=4$ is hypergeometric: [c]25:207, [c]33:68.

D(midrange): [d]21:100.

D(range): [c]17:364, [c]18:173, [c]24:404, [c]39:130.

Quasi-range: [d]28:179.

D(r): [b]15:193.

$D(\xi)$: [d]26:114.

D(Q): *MR22:60.*

$D(x_1x_2)$: [d]7:1, [d]18:265.

$D(x_1, \ldots, x_n)$: [4]98.

$FD(m) = N(\bar{x}, vn^{-1})$: [c]30:401, 414, [p]17:231.

$FD[n^{\frac{1}{2}}s^{-1}(\bar{x}-m)]$ = Student $(n-1)$: [3]88.
Tests: [3]98, [c]33:173, [n]5:90, [d]9:279, [k]6:395.

FD(m, σ): [3]89, [k]6:395, [d]10:68.

$FD(1/\sigma)$ = Helmert $(n-1, 1/s\sqrt{n})$: [3]89.

A priori distributions of m and $1/\sigma$: *MR9:48.*

Ranking means: [t]7:131.

C. *Estimation*

$C.-R.(m) = vn^{-1}$, i.e. $\bar{x}$ efficient: [1]483, [4]135, [3]20, [y]11:182.

$Var(\bar{x}) \leq var(\xi)$: [4]92.

$UMVUE(m) = \bar{x}$: [3]51, [t]4:167.

$\bar{x}$ unbiased: [p]7:150.

Minimax estimates of m: [d]21:218, [d]22:28.

Best "density unbiased" estimate of m: [d]25:399.

~efficiency of ξ is $2/\pi = .6366$: [1]490, [u]22:706.

Estimation of m when it must be integral, etc: [b]12:192.

Mean of k^{th} values from top and bottom has ~efficiency zero: [1]490.

$C.-R.(\sigma) = v(2n)^{-1}$, hence s is efficient: [2]224.

Efficiency of estimates of σ: [c]37:182.

In estimating σ,

$\sqrt{\frac{1}{2}n}\frac{\Gamma(\frac{1}{2}n)}{\Gamma[\frac{1}{2}(n+1)]}\sqrt{\frac{1}{n}\Sigma(x_i-m)^2}$ is more efficient than

$\sqrt{\frac{1}{2}n}\,\frac{\Gamma[\frac{1}{2}(n-1)]}{\Gamma(\frac{1}{2}n)}\,s$: [1]484.

C.-R. (σ/m): [c]8:204.

$[n/(n-1)]s^2$ unbiased for v: [p]7:152.

Estimation of σ for industrial quality control: [g]49:375.

Estimation of σ from percentiles: [i]40:85.

Estimation of v and σ: [d]18:584, [b]1:78.

$C.-R.(v) = 2v^2n^{-1}$, hence s^2 efficient: [1]484.

$MLE(v) = s^2$: [6]10.3.

MLE(v) depends on whether m known: [3]34.

Estimation of v: [e]12:57, [i]40:85.

In estimating v, ~efficiency of $s^2 = +1$: [3]7.

If m constant, s^2 not sufficient: [3]11, [4]136.

In estimating v, ~efficiency of mean deviation = .876: [3]7.

Unbiased estimates: *A. M. S. Translation No. 98.*

Closest estimates of m,v: [u]33:45.

Unbiased estimation of mean absolute deviation: *MR13:367.*

In estimating m,v ~variance-covariance matrix: [4]142.

$\bar{x}$, s^2 are moments estimates of m,v: [1]498.

$\bar{x}$, $[n/(n-1)]s^2$ are ~efficient: [1]494.

$MLE(v,m) = \bar{x}$, s^2: [1]504, [6]156, [3]20, 34, [4]132.

$\bar{x}$, s^2 joint sufficient: [3]11, [4]136, [e]17:211, [i]38:181.

Estimation of m, σ: [d]17:386, [e]10:321, [e]8:12.

Estimation: [c]35:186.

Minimum χ^2 estimation: [c]11:262.

MLE from censored sample: [i]32:124.

Censored sample: [c]39:260, [c]43:225, *MR14:569, MR15:241.*

Estimation of m^2: [u]45:214, [d]17:43

If v known, confidence regions for m are $\bar{x} \pm k_p v^{\frac{1}{2}} n^{-\frac{1}{2}}$, where k_p are the p% values of the normal: [1]514.

Confidence intervals for m: [4]130, [6]224, [10]189.

Seq. confidence intervals for m: [d]18:427, [b]19:133.

Confidence intervals for v: [4]131, [6]226.

Interval estimation of v and σ: [e]6:117.

Confidence limits for v_1/v_2: [4]131.
Confidence limits for m_1-m_2 with same v: [4]130.
Interval estimation of m_1-m_2 (Behren's Problem): [3]91, [d]18:601, [d]14:35, [d]15:430, [d]20:616, [d]21:507, [e]4:39, 108, [d]24:390 [p]7:232.
Confidence limits on m and s: [e]2:13, [o]8:83.
Confidence intervals for m_i v: [6]227, [3]79.
Tolerance limits: [d]13:398, [d]17:208, (~) [d]17:238, [d]27:171, [t]1:164.

D. *Testing*

Testing $m_1>m_2$: [d]14:149.
Tests on m: [6]259.
Unbiased regions for testing $m_1=m_2$: [3]320.
Power function of $m\geq m_0$: [3]305.
Test of m using range in place of s: [d]17:71.
Hypotheses on m: [1]533, [4]149.
Tests based on (rectangular a priori) distributions of m and v: Z18:158.
Seq. tests on m: [d]16:171, [e]10:364, 368, [b]9:250, [c]37:334.
Control charts on m: [b]16:131.
Testing m_1 against m_2 and σ_0 against σ_1 by quick counting methods: [e]17:80.
Seq. hypotheses on m: [d]20:502, [g]40:303.
Student's hypothesis: [g]31:318.
Student's is best for testing $m_1=m_2$: [3]285, 291, [e]12:79.
LR test of $m=m_0$ is Student: [4]150.
Power function for Student test: [d]17:192.
Seq. Student test: [c]37:326.
Comparison of two means: [10]190, [c]38:252, [c]35:88, [o]4:31.
Comparison of k means: [t]7:1.
Testing whether variance is constant: [y]20:114.
Three decision seq. test of m: [y]23:22, *Amsterdam Mathematical Centre Stat. Dep. Rep SP34.*
Testing whether many means are all zero: [e]8:70, [4]176.
Testing $\bar{x}_1-\bar{x}_2$: [i]29:21, [c]41:361.
Testing $\bar{x}_1-\bar{x}_2$ without assuming $v_1=v_2$: [d]9:201, [13]433.
Linear hypotheses: [c]27:161, [3]292, 300.
Joint tests: [t]6:25, 73.
Testing outlying observations: [e]17:67.
H: $\sigma=\sigma_0$: [3]287, [d]8:193.
OC for χ^2 test of $\sigma=\sigma_0$: [d]17:179.
Seq. test on σ: [e]10:369.
Seq. test on $\sigma_1=\sigma_2$: [e]12:63, [b]11:101.
Tests on v: [6]267.
Fisher is best for testing $v_1=v_2$: [3]289.
Testing homogeneity of variances: [c]31:250.
The most powerful test of

$$\left\{\begin{array}{c}\text{H: } \sigma=\sigma_0,\ m\\ \text{Alt: } \sigma=\sigma_1,\ m=m_1\end{array}\right\} \text{ is } \Sigma(x_i-\bar{x})^2 \gtrless c.$$

Hypothesis of equality of many normal variances: [b]6:89, [c]29:124.
Significance of smallest of set of variances: [s]10:117.
Critical regions for m and v: [3]278.
Bibliography of testing equality of variances: [a]109:457.
Seq. ratio test terminates: [e]8:342.
OC function: [g]47:191.
Power functions of tests: [d]17:189.
Whether two samples are from the same normal population: [n]7:3, [k]17:302.
Tests for normality: [c]28:295, [c]27:310, 333, [c]34:209, [o]1:125, [i]20:152, Z19:74.
Tests of various composite hypotheses: [d]19:495, [e]9:30, [e]10:29.
Decision problems: [b]15:55.

E. *Miscellaneous*

Independence of x and s (Student-Fisher Theorem): [i]19:108, *J. Math. Soc. Jap. 1:111*, MR1:346.
s^2, $\bar{x}$ independent: [4]108, MR14:775.
Normality if and only if $\bar{x}$ and s^2 independent: [d]13:91, [d]16:400, [d]13:91, *NBS Rep. 2267*, *J. Math. Soc. Jap. 1:111.*
Normality if and only if $D(\bar{x}, s)\equiv L\cdot s^{n-2}$: [d]14:197.
Normality if $D(x)\,D(y)=\phi(\sqrt{x^2+y^2})$: MR10:125.
Various characterizations of normality: [e]13:359, [d]28:126, [i]39:59, [d]27:858, [e]14:180, *Am. J. Math. 61:726*, *Math. Z. 41:405*, Z13:214, *3rd Berkeley Symp. 2:195*, MR16:1034.
Generalization of Student-Fisher theorem: [i]20:248.
Independence of quadratic forms: [o]1:83, *Proc. Roy. Soc. Edin. 60:40*, [u]30:178, [c]37:93, [c]14:195, [d]15:427, [d]20:119, MR12:509.
Bayes' theorem: [n]16–1:113.
Cochran's theorem: [4]107,68,
More generally: [d]11:100.
History of normal: [c]16:402.
History of distribution of s: [c]23:416.
Distributions which converge to normality: [d]10:247.
Discrete analog: [c]44:365.
Regression of x and t, where $m=a-be^{-kt}$: [d]18:596.
Comparing percentage points of two normals: [d]19:93.
k samples: [3]295.
Truncated sample: [d]23:237.
Sampling from $N(\Sigma\, a_k m_k, v)$: [4]160.
Max Σx_i, min Σx_i: [c]40:35.
See also: [d]25:389, [d]1:151, [d]6:197, [d]7:77, [d]10:365, [d]13:235, [d]17:483, [d]20:123, [d]21:362, [d]21:557, [d]22:596, [d]23:43, [d]23:384, [d]23:547, [d]25:16, [l]3:309, [e]9:6, [13]345, [c]10:522, [c]13:287, [c]16:239, [c]24:184, [a]79:455, [f]7:23, [c]31:238, [c]34:61, 98, [n]5:3, [n]12-3:65, [c]40:116, [c]32:226, 301, [g]48:550, [d]25:636, 698, [e]5:337, [g]51:88, *Brit. Assoc. Math Tables (3rd Ed.) v.I p. xxviii*, *N.B.S. Rep. 2545*, *C.R. Acad. Sci. Paris 238:444*, [o]1:83, *App. Sci. Research (Netherlands) Sect. A 3:297*, *J. Franklin Inst. 260:209*, *N.B.S. Rep. 2267*, *Am. J. Math. 57:821*, *Ann. Math. 35:312*, *Nat. Acad. Sci. 28:297*, [y]24:2, Z18:225, MR16:52, MR14:1098, Z5:173, Z19:317, MR17:53, [w]7:193.

1.2. Normal: N (0, v)

$D(x) = (2\pi v)^{-\frac{1}{2}} \exp(-\frac{1}{2} x^2 v^{-1})$: *[10]50, [c]31:1.*
$Ch(x) = \exp(-\frac{1}{2} vt^2)$: *[2]94.*
$MGF(x) = \exp(\frac{1}{2} vt^2)$: *[2]53.*
$C(x)$: *[c]25:379.*
$\alpha_{2k} = \sigma^{2k}(2k)!/2^k k!$: *[10]54.*
2nd cumulant = v, others zero: *[2]67.*
Pearson type: *[2]141.*
$D(\bar{x}) = N(0, v/n)$: *[2]175, [n]10–3:90.*
$D(x/y)$ = Cauchy: *[18]1–150, [w]1:74.*
$D(x^2)$ = Type III: *[i]26:212.*
$D(s^2)$: *[2]246, [u]28:456, Z15:118.*
$D(\Sigma x_i^2/2v)$ = Gamma $(\frac{1}{2} n)$.
$D(\Sigma x_i^2)$ etc: *[c]35:47.*
$D(\Sigma x_i^2)$, $D(Q_1/Q_2)$: *[w]1:74.*
$D(Q/v) = \chi^2(r)$, Q of rank r; $\neq 0$ eigenvalues all $+1$: *[d]9:48, [c]25: 122. Z19:357 [c]30:407.*
$Ch(Q) = [\Pi (1-2itvk_i)^{\frac{1}{2}}]^{-1}$.
$D[x\, n^{\frac{1}{2}} (\Sigma x_i^2)^{-\frac{1}{2}}]$ = Type II: *[i]29:13.*
D(x) assuming v is Type III: *[d]28:510.*
D(x/s) = Student (testing): *[c]37:65.*
D(range) for n = 3, unbiased critical region: *[3] 327.*
$FD(v)$ = Type V$[\Sigma x_i^2/(n-2), \frac{1}{2}(n-2)]$: *[p]7:226.*
C.–R. $(v) = 2n^{-1}v^2$: *[1]484, [p]7:159.*
If D(n) = No. 3.5: *MR14:391.*
$MLE(v) = n^{-1}\Sigma x_i^2$: *[4]141.*
s^2 UMVUE of v, but s not of σ: *[3]52,54.*
Neyman-Pearson on hypothesis testing: *[c]20: 178.*

Most powerful test of $\left\{\begin{matrix} H: \sigma = \sigma_0 \\ Alt: \sigma = \sigma_1 \end{matrix}\right\}$ is $\bar{x}^2 + s^2 \lessgtr c$: *[3]275.*

Completeness: *[e]10:313.*
Unbiased critical regions: *[3]212.*
Testing s_1^2/s_2^2 etc: *[e]5:157.*
Testing serial correlation: *[i]31:103.*
Various devices for showing area $= +1$: *[d]5:136.*
Inference: *[b]15:52.*
As "Maxwell-Bolzmann" distribution: *[12]39.*
Variance of Gini's mean difference is $\sim v n^{-1}(.8068)^2$: *[2]217.*
Mean difference: *[c]28:432.*
Properties of f(x), where x is N(0,v): *[c]17:211.*

See also: *[d]12:239, [c]3:311, [g]26:178, [c]31: 260, [n]13–1:51, [u]30:330, MR9:364, MR3:2, [a]83: 127.*

1.3. Normal: N(m,1)

$D(x) = (2\pi)^{-\frac{1}{2}} \exp[-\frac{1}{2}(x-m)^2]$.
$E(x_1^2 + x_2^2) = (\frac{1}{2}\pi)^{\frac{1}{2}}$, $Var(x_1^2 + x_2^2) = 2 - \frac{1}{2}\pi$, $E|x| = (2/\pi)^{\frac{1}{2}}$, $E(e^{ax}) = e^{\frac{1}{2}a^2}$, $var(e^{ax}) = e^{2a^2} - e^{a^2}$: *[8]120.*

$D(\bar{x})$: *[3]2.*

$D(x^2)$, $D(x_1^2 + x_2^2)$, $D(x_1^2 + x_2^2)^{\frac{1}{2}}$: *[8]95.*

D(xy) = Bessel: *MR10:200.*

$FD(m) = N(\bar{x}, n^{-1})$: *[3]85.*

Bayes' distribution (m): *[3]91.*
$\bar{x}$ sufficient: *[3]8, [e]17:211, [p]7:161.*
$\bar{x}$ is consistent: *[3]3,26.*
$\bar{x}$ is MLE: *[4]140, [c]33:125.*
$\bar{x}$ is minimax.
Efficiency of $\xi = .637$: *[3]6.*
Confidence intervals for m: *[3]63,70, [p]7:222.*
MLE for χ^2 test: *[d]25:580.*
Remarks on testing: *[d]13:62.*

Most powerful test of $\left\{\begin{matrix} H: m \leq m_0 \\ Alt: m = m_1 m_3 \end{matrix}\right\}$ is $\bar{x} > c$: *[3]274.*

UMP test of m = 0: *[13]452.*
Seq. test: *[c]43:452.*
Completeness: *[e]10:313.*
Unbiased critical regions: *[3]311.*
Testing equality of several means: *[e]8:69.*
Testing; called "Laplace": *[c]2:251.*
Peculiar composite hypothesis on m: *[3]306.*
Inference: *[b]15:52.*
Pitman's method: *[3]324.*
Range: *[c]38:463.*

See also: *[c]27:466, [c]31:202, [c]36:460, [g]7:95 3rd Berkeley Symposium 1:197.*

1.4. Normal: N (0, 1) (Gaussian)

$D(x) = (2\pi)^{-\frac{1}{2}} \exp(-\frac{1}{2}x^2)$: *[7]129.*
General expose: *Acta Math 77:1.*
C(x) as continued fraction: *[2]130.*
C(x) as a series: *[c]19:13.*
Bounds on C(x): *[c]42:263.*
Property of C(x): *Math. Zeit 41:405.*
C(x): *MR10:267.*
$\sim C(x)$: *MR16:628.*
$MGF(x) = \exp(\frac{1}{2}t^2)$.
$Ch(x) = \exp(-\frac{1}{2}t^2)$: *[8]164, [1]100.*
$\alpha_{2k} = (2k)!/2^k k!$: *[d]5:32, [d]11:353, [h]1:13,193, [1]208.*
Absolute moments: *Z1:26.*
Median and quartiles: *[c]25:79.*
$D(n\bar{x})$ = Normal: *[c]19:227.*
$D(s^2)$: *[e]5:138.*
$D(x^2)$: *[d]1:340, [e]5:138.*
$D(\Sigma x_i^2) = \chi^2(n)$: *[2]231, [4]103, [9]331.*
$D(\Sigma k_i x_i)$: *[d]13:17.*
$C(n^{\frac{1}{2}} s^{-1} \bar{x})$ = Student: *[9]336.*

$D(Q)$, $D(Q_1/Q_2)$: [e]17:37.

$D[\Sigma(x_1-\bar{x})^2]=\chi^2(n-1)$: [9]333.

C(r) expressed as an integral: [9]339.

$D(x_1x_2)$ by Mellin transformation: [d]19:375.

$D(x_1/x_2)$=Cauchy: [d]19:375.

$D(\bar{x},s)$: [o]7:65.

D(range) for n=3, ~D(range): [c]34:111, [c]5:313, [o]8:155, [c]36:142.

Moments of sample median: [d]26:600.

D(extreme deviate): [c]35:120.

C(range): [c]32:341.

$D(\Sigma x_i^2/\Sigma y_i^2)$=Snedecor: [d]19:378.

Ch fcns of estimates of v: [d]19:257.

Generating functions: Z2:200.

Estimation of dispersion: [c]36:96.

Estimation of mean deviation: [c]33:254, [c]35:304.

Variance of median: [c]23:361.

Testing N(0,1) against various alternatives: [c]30:139.

Multivariate analysis: [3]XXVIII.

Limit of binomial: [7]134.

Central limit theorem: [i]27:139, [i]29:206

K^{th} value from the top: [1]374.

Censored samples: [c]41:230.

Ordered samples: [e]11:23.

Stratified sampling: [d]5:138.

Variance in two samples: [n]13–3:49.

Ratio of two ranges: [d]21:112.

Tetrachloric functions: [c]14:157.

Approximations: [d]17:363.

Sheppard's tables: [c]2:174.

Grouping: [i]32:135.

Moments of order statistics: [d]41:200.

Occasionally called Laplace-Gauss, or even Laplace: *Acta Math. 77:1, C. R. Acad. Sci. Paris 232:1999.*

See also: [d]4:109, [d]17:350, [d]22:425, [d]24:133, [d]24:297, [13]63, [e]18:395, [c]24:98, [c]24:280, [c]25:195, [i]6:209, [17]No. 41, [m]6:120, [d]22:418, [y]24:22, [u]29:231, Z4:66, Z8:266, Z20:39, 145, MR12:191, Z18:412, MR17:756, C. R. Acad. Sci. Paris 238:444, Phil. Trans. Roy. Soc. London A237:231, MR7:18, Z5:366, [y]4:189.

1.5. Truncated Normal

C(x): [1]248, [6]243.

Introduction, estimation, examples: [15]144.

$D(\Sigma x_i)$: [b]8:223.

Fitting: [c]39:252.

Estimating m and v: [g]44:518, [g]47:457, [f]9:489, [o]3:37, MR7:461 [c]40:52.

Distribution of estimate of σ: [15]316.

Censored sample: [c]42:516.

MLE: [i]32:119.

See also: [d]9:66, [d]20:458, [g]47:379, *Brit. Assoc. Math. Tables (3rd Ed.) v. 1 p. xxxv, MR2:231.*

1.6. Generalized Normal (Kapetyn)

$(2\pi v)^{-\frac{1}{2}}\exp[-\frac{1}{2}v^{-1}(f(x)-m)^2]df(x)$: [5]93.

C.–R.(m)=v/n, C.–R.(v)=$2v^2/n$: [5]139.
MLE(σ)=$[n^{-1}\Sigma(f(x_i)-m^2]^{\frac{1}{2}}$,
MLE(m)=$n^{-1}\Sigma f(x_i)$.

See also: [c]5:168.

1.7. Normals Added

$D(x)=(1+k)^{-1}\{(2\pi)^{-\frac{1}{2}}\exp(-\frac{1}{2}(x+m_1)^2+kv^{-\frac{1}{2}}(2\pi)^{-\frac{1}{2}}\exp-\frac{1}{2}(x-m_2)v^{-1}\}$: [d]2:340, [h']2:63.

Var $(x)=(1+k)^{-1}[1+m_1^2+k(v+m_2^2)]$.

Method for partition with example: [s]5:47.

Semi-invariants: [i]17:1.

$D(\bar{x})$: [d]11:219.

Three normals added: [d]5:237.

More generally: [d]3:1, [d]5:230, [c]3:85, MR14:485.

Sampling theory: [n]8–3:67.

Many normals added: [c]37:429.

Called "compound normal": [i]32:180.

Bivariates: [f]8:328.

See also: [c]40:460, [e]14:369, MR11:258, MR17 1102.

1.8. Lognormal (a, m, v)

$D(x)=(x-a)^{-1}(2\pi v)^{-\frac{1}{2}}\exp[-\frac{1}{2}v^{-1}(\log(x-a)-m)^2]$, parameters and moments: [15]160, [5]121, [d]3:45. [1]258, [c]4:194.

Graphical determination of parameters: [w]9:102.

Mean=$a+e^{m+\frac{1}{2}v}$, var=$e^{2m+v}(e^v-1)$.

Another form

$$D(x)=\frac{1}{\sqrt{2}\,c(x-a)}\exp\left\{-\frac{1}{2c^2}\left[\log\frac{x-a}{b}\right]^2\right\}$$

$m=be^{\frac{1}{2}c^2}+a$, mode=$be^{-c^2}+a$, GM=ξ =$a+b$, moments, tables, regression, examples, bibliography: [d]4:30, [b]7:155.

Moments, transformations: [w]7:152, [w]8:83.

Complete treatment with bibliography: *Aitchison, J. and Brown, J. A. C., "The Lognormal Distribution" Cambridge, 1957 (MR18:957).*

Estimation of m: [e]10:341.

MLE: [g]46:206, *Intl. Congr. Math (1950) 1:581.*

Regression: [d]7:196.

~Tests on m: [d]28:1044, [d]27:670.

Called "Galton-Macalister": [c]32:239.

Called Gibrat: *Kendall and Buckland, "A dictionary of Stat. terms."*

Used to approximate Fisher distribution: [d]12:448.

Deduced from hypothesis about errors, etc.: [i]28:141.

$(x-a)/(b-x)$ lognormal: [17]No.46.

Transformation: [c]36:155.

Versus normal: *Geochimica et Cosmochimica Acta 8:53.*

Discrete lognormal: [c]37:362.

Compared with normal by means of Galton-Kapetyn apparatus: [s]4:129.

Truncated lognormal: [i]28:150, [c]38:414.

Lognormal (0, 0, 1), $E(x)=e^{\frac{1}{2}}$, $v=e^2-e$: [8]120, 176, [17]No. 45, [c]22:109, [d]4:30.

See also: [d]14:120, [1]13:161, [e]12:121, [b]6:174, [b]11:19, [d]15:182, [c]4:179, [c]22:146, [g]34:762, [g]36:493, [f]1:57, [c]36:155, [c]38:427, [g]48:600, *J. Franklin Inst.*, 244:471, 250:339, 250:419, 251: 499, 251:617, [g]50:904, [c]43:404, [a]119:157, 185, 250:339, *Ind. and Eng. Chem.* 40:2289, *J. Roy. Soc. (A)*216:309, *J. Phys. Chem.*, 56:442, [y]13:29, *J. Hygiene* 42:328, Z10: 173, MR3:4, *Nature* 156:463.

1.9. Wrapped-up Normal

$D(x)=k\Sigma e^{-c(x+1)^2}$: *Bull. Soc. Math. France 66: 32, 67:1, C. R. Soc. Math. France (1938) p 34, Ann. Ecole. Norm. Sup. 45:1* [t] 55:335, [d]18:589, *Handbuch der Physik, Berlin, Springer 3:477.*

1.10. Gram-Charlier

Two-term $D(x)=(2\pi)^{-\frac{1}{2}}[1-k/6(3x-x^3)]\exp(-\frac{1}{2}x^2)$: [3]103, 137.

General Gram-Charlier: [4]77.

$D(\bar{x})$: [d]1:199, [d]2:99.

$D(x^2)$: [i]26:212.

D(s): [d]6:127.

t-test: [i]26:210.

$D(x)=f(x)N(m,v)$: [d]23:467, [g]26(P):233.

D(various statistics): [g]4:1.

Log Gram-Charlier: [i]28:145.

Type B Gram-Charlier: [d]8:183, [d]18:574, *Trans. Amer. Math. Soc. 67:206*, [d]20:376, [i]5:17.

MGF factorial moments: Z5:213.

See also: [a]88:576, [a]89:129, [c]33:126, [c]36: 427, [c]38:58, 87, [c]39:425, [i]7:147, [l]23:283, *T.A.M.S. 67:206*, Z18:320, Z22:243, Z2:43

1.11. Bivariate Normal $N\left(\begin{smallmatrix}m_1, & v_1\\ m_2, & v_2\end{smallmatrix}\ \rho\right)$

$D(x,y)=[2\pi\sigma_1\sigma_2(1-\rho^2)^{\frac{1}{2}}]^{-1}\ \exp\{-\frac{1}{2}[v_1v_2(1-\rho^2)]^{-1}[(x-m_1)^2v_2-2\rho\sigma_1\sigma_2(x-m_1)(y-m_2)+(y-m_2)^2v_1]\}$ [6]165, [5]89, [4]60.

Introduction, properties, examples: [15]585.

Another form (Koopman-Darmois): [e]8:322.

$Ch(x,y)=\exp\{i(m_1s+m_2t)-\frac{1}{2}(v_1s^2+2\rho\sigma_1\sigma_2st+v_2t)\}$: [1]287.

MGF: [6]167.

$D(\bar{x}_1,\bar{x}_2)$ = Normal bivariate: [4]101.

$D(\bar{x}-\bar{y})$: [c]2:379.

D(y/x): [c]24:428.

D(xy) and D(x/y): [18]1–151, *Am. Math. Monthly* 49:26.

Ch(xy): [a]42:82.

D(r): [4]120.

If $\rho=0$, $D(r^2)=B(1, n-2)$: [10]160, 178.

If $\rho=0$, $D(b)=B(1, n-1)$: [10]180.

D(correlation ratio) = No. 5.3: [10]181.

~D(r): [c]10:507.

$D(rs_1s_2, s_1^2, s_2^2)$: [u]29:264.

Distribution of various statistics: [e]17:21.

$MLE(m_1,m_2,v_1,v_2,\rho\sigma_1\sigma_2)=\bar{x},\ \bar{y},\ s_1^2,s_2^2,rs_1s_2$: [3]37.

$Var(\bar{x})=v_1/n$, $var(s_1^2)=v_1/2n$, $var(r)=n^{-1}(1-\rho^2)^2$, $cov\ (s_1,s_2)=\rho^2\sigma_1\sigma_2/2n$, $cov(r,\ s_1)=\rho\sigma_1(1-\rho^2)/2n$, $cov\ (\bar{x},\bar{y})=\rho\sigma_1\sigma_2/n$: [3]38.

MLE from fragmentary information: [d]3:163.

$\bar{x}$, $\bar{y}$ are joint efficient in estimating m_1 and m_2; $\bar{x}$, $\bar{y}$, $\frac{ns_1^2}{n-1}$, $\frac{ns_2^2}{n-1}$, $\frac{n}{n-1}rs_1s_2$ have joint efficiency $(n-1/n)^3$: [1]495–6.

$[(x-m_1)/\sigma_1]+[(y-m_2)/\sigma_2]$ and $[(x-m_1)/\sigma_1]-[(y-m_2)/\sigma_2]$ are independent and $N(0,2(1+\rho))$, $N(0,2(1-\rho))$ respectively: [6]217.

Estimation: [d]17:395, [e]8:322.

Estimation, testing: [15]606, [g]50:884.

Censored samples: [x]6:83.

Distribution ratio standard deviations: [x]6:93.

Confidence limits for r: [c]29:157, *J. Nat. Inst. Personnel Res. 6:153.*

Confidence limits for m_1/m_2: [d]13:440, MR13: 962.

Sufficient statistics: [e]17:212.

Comparison of two correlations: [10]203.

Tests of seven hypotheses on the parameters: [d]11:410.

Testing equality of two r's: [d]12:279.

Some tests: [w]7:46.

Testing equality of variances: [e]*1:13, (bibliography)*, [a]*109:462*.

r_1-r_2: [c]*25:102*.

Sequential tests of ρ: [w]*8:202*.

Truncation: *MR2:231*.

Fisher's original work on r and ρ: [n]*1–4:1*.

Sufficient conditions for normal bivariate: [e]*6:399, MR15:805*.

k samples: [e]*27:145, 227*.

$x/\sigma_1+y/\sigma_2$ and $x/\sigma_1-y/\sigma_2$ are independent normal variables: [c]*31:9*.

If $\rho=0$ then $(v_1/v_2)/[(v_1/v_2)+(s_1^2/s_2^2)]$ is Beta: [c]*31:10*,

Called Bravais distribution: [i]*19:3*

Properties: *Z15:310, MR1:246*.

See also: [d]*4:196*, [d]*14:141*, [e]*7:260*, [g]*26:129*, [c]*22:1*, [c]*25:356*, [c]*25:392*, [f]*8:328*, [n]*9–3:90*, [c]*39:238*, [i]*27:221*, [6]*218*, [c]*5:311*, [d]*27:1075*, [c]*44:289*, [x]*4:85*, *Harvard Ed. Rev. 1946, p. 52*, *MR4:280*, *MR8:283*, *MR14:1102*, *MR7:212*, [y]*1(No. 4) 20*.

1.12. Bivariate Normal $N\left(\begin{smallmatrix}0,&v_1\\0,&v_2,\end{smallmatrix}\ \rho\right)$

$D(x,y)=(2\pi M^{\frac{1}{2}})^{-1}\exp[-\frac{1}{2}M^{-1}(x^2v_2-2\rho\sigma_1\sigma_2xy+y^2v_1)]$

where $M=\begin{vmatrix}v_1 & \rho\sigma_1\sigma_2\\ \rho\sigma_1\sigma_2 & v_2\end{vmatrix}=v_1v_2(1-\rho^2)$; $Ch(x,y)=\exp(-\frac{1}{2}n^{-1}(v_1s^2+2\rho\sigma_1\sigma_2st+t^2v_2))$]: [9]*308*, [2]*22, 334*, [4]*60*, [8]*116*, [15]*588*, [10]*95, 106*, [c]*30:8*.

$\alpha_{40}=3v_1^2$, $\alpha_{31}=3\rho\sigma_1^3\sigma_2$, $\alpha_{22}=(1+2\rho^2)v_1v_2$: [2]*80*, [4]*60*.

Moments: [o]*3:2*, [c]*12:177*.

Central moments: [h']*4:73*.

Incomplete moments: [c]*13:401*, [c]*40:22*.

Product-moments: [c]*12:86*.

As limit of binomial: [a]*91:548*.

If $\rho=0$, $\sigma_1=\sigma_2$ called "circular normal," D(r), properties: [c]*29:137*.

C(x,y) with other properties: [c]*33:59*, [c]*38:475*.

Cumulants in terms of moments: [2]*89*.

$Var(r)=n^{-1}(1-\rho^2)^2$: [2]*336*.

$Var(b)=n^{-1}v_1^{-1}v_2(1-\rho^2)$: [2]*337*.

Marginal and conditional distributions: [4]*62*.

Regression: [3]*144*.

Correlation and regression with generalization: [i]*24:1*.

Contour ellipses: [s]*1:203*.

Bilinear forms: [o]*1:103*, [d]*18:565*.

Quadratic forms: [d]*14:195*.

$Ch(x^3,y^2)$: [2]*336*.

D(variance-covariance)=Wishart: [2]*340*, [1]*29.6*.

D(r): [2]*342*.

D(r) for n=4: [u]*26:536*.

D(b)=Type VII: [1]*402*.

$D(s_1/s_2)$=No. 8.3 if $\sigma_1=\sigma_2$: [e]*2:65*, [c]*31:9*.

$D(e^x,e^y)$: [w]*2:155*.

When, further, $\rho=0$ generalized Student: [c]*30:190*.

Simple function of x/y is normal: [a]*93:442*.

D(y/x): [c]*32:16*, [i]*20:61*.

D(xy), D(x/y): *MR3:171*.

Joint distribution of Pearson betas: [c]*7:386*.

$D(s_1^2, s_2^2, rs_1s_2)$ etc: [d]*5:283*.

$D(s_1^2, s_2^2)$: [e]*5:139*, [c]*25:126*.

$$D\left(\frac{(n-1)^{\frac{1}{2}}(b-\beta)\sigma_1}{\sqrt{(v_2-b^2v_1)}}\right)=\text{Student }(n-1).$$

$$D\left(\frac{(n-2)^{\frac{1}{2}}(b-\beta)s_1}{\sqrt{v_2-bv_1}}\right)=\text{Student }(n-2)\text{: [9]}5.13\text{, [2]}348\text{, [3]}156.$$

Moments of dist. of covariance from

$N\left(\begin{smallmatrix}0,&1\\0,&1,\end{smallmatrix}\ \rho\right)$: [3]*334*.

D(radial standard deviation)=$D[n^{-1}\Sigma(x_1-\bar{x})^2+\Sigma(y_1-\bar{y})]^{\frac{1}{2}}$: [d]*15:75*.

D(sample covariance): [3]*359*, [10]*138*.

MLE(ρ): [3]*33*.

Confidence intervals for ρ: [3]*81*.

$MLE(v_1,v_2,\rho)$: [2]*339*.

Estimation of ρ when $v_1=v_2$: [d]*9:149*.

Estimation of ρ by rank correlation: [d]*7:40*, [c]*40:419*.

Truncation: [d]*21:272*.
and estimation: [e]*12:277*, *MR16:498*.

Testing v_1/v_2: [3]*138*.

Testing σ_1/σ_2 and ρ: [c]*5:151*.

Test whether two samples are from same population: [3]*140*.

Hotelling's generalized T applied to tolerance limits: [d]*14:90*.

Odd fact for $\rho=0$: [d]*18:442*.

See also: [o]*12:90*, [c]*2:369*, [c]*4:498*, [c]*17:176*, [c]*20:295*, [g]*27:254*, [a]*83:128*, [c]*29:74*, [n]*2:640*, [n]*7:6*, [c]*32:196*, [c]*38:371*, [i]*7:220*, [i]*24:17*, [n]*13–1:21, 65*, [u]*28:457*, *Z12:267*.

1.13. Trivariate Normal

D(x): [10]*255,260*.

Moments: [o]*4:15*, [c]*40:23*.

Correlation: [i]14:158.

Partial correlation: [c]10:391.

Yielding 2×2×2 table: [c]10:272.

Student test for partial correlation: [10]256.

Snedecor test for multiple correlation: [10]257.

See also: [d]8:179, [d]12:94, [n]1-1:151, [u]44:342, MR15:805.

1.14. Multivariate Normal

$D(x)=Ce^{-Q}=[(2\pi)^{\frac{1}{2}n}\sigma_1 \ldots \sigma_n]^{-1}(|A|^{\frac{1}{2}})\exp\{-\frac{1}{2}\Sigma\Sigma(a_{ij}/\sigma_i\sigma_j)(x_i-m_i)(x_j-m_j)\}$ where $A=||a_{ij}||=||\rho_{ij}||^{-1}$ =inverse of correlation matrix: [6]177, [2]376, [4]63.

C(x): [c]40:458, [c]41:351.

$Ch(x)=\exp\{i\Sigma m_j t_j-\frac{1}{2}\Sigma\Sigma\rho_{ij}\sigma_i\sigma_j t_i t_j\}$.

Moments: [c]40:20, MR5:42.

Marginal distributions, conditional distributions, regression: [4]70.

Independence of quadratic forms: [o]1:83.

Distributions of moments, partial and multiple correlations: [i]24:185, [i]27:235, [i]28:20.

D(product moment)=Wishart: [c]20:32, [i]20:218, [u]35:336, [u]29:260, 271, MR10:387, [b]17:82.

Cumulants of logarithmic generalized variance: [i]38:17.

Independence of distribution of means and second order moments: [4]233.

D(multiple correlation): [d]3:196.

Fiducial distribution: [u]34:41.

Multiple and partial correlation: [c]19:100.

Linear regression theory: [4]245.

$D(Q)=\chi^2$: [4]104.

D(various Q): MR13:142.

D(vector correlation): [c]28:353.

Sampling: [4].XI, [d]6:202.

$MLE(m_1, \ldots, m_n)=(\bar{x}_1, \ldots, \bar{x}_n)$: [6]187.

Hotelling's generalized Student test: [4]234, [d]9:240.

LR test for variances equal and correlations zero: [d]11:204.

LR test for independence of variables: [d]11:17.

Characterization: [e]14:367.

Hypothesis of equality of means: [4]238.

Independence of sets of variables: [4]242.

Probability that all n variables are positive: [d]26:484.

Tolerance regions: [d]27:174.

Testing variance-covariance homogeneity: [c]31:31, [c]34:311.

Truncation: [x]5:17.

Various tests: [d]17:257, [d]21:293.

Quadrivariate: [c]43:206.

Bibliography of tests of hypothesis of equality of variances called 'Bipolar' distribution: [e]14:61.

Variables separated into two sets: [c]30:295.

Many samples: [c]31:221.

Seq. analysis: [e]12:328.

Central limit theorem: [i]28:109.

Generalizations of N(m,v) theorems: [e]17:221.

See also: [d]8:149, [d]17:344, [d]19:447, [d]21:445, [e]3:273, [e]12:99, [e]6:35, [a]90:136, [c]6:59, [c]15:192, [c]35:58, [o]1:79, [b]18:70, [c]43:212, [x]1:59, [t]6:181, [g]52:200, MR13:366, MR17:278, MR12:345, MR15:141, MR6:159, Z10:406, Z15:220, MR10:312, Trans. Am. Math. Soc. 24:135.

2. Type III Distributions

2.1. Type III (p, q)

$D(x)=\frac{p^q}{\Gamma(q)}x^{q-1}e^{-px}, (0, \infty)$

$D(x)=\frac{x^b e^{-x/a}}{b!\,a^{b+1}}$ ("gamma"); [6]112.

D(x)=Type III (k/a, k) ("Erlang"): [d]24:339

D(x)=Type III (p/q, p) ("Eulerian"): [c]35:6.

$Ch(x)=(1-it/p)^{-q}$: [2]55, [1]126.

$MGF(x)=(1-at)^{-b-1}$: [6]115, [4]74.

$\alpha_1=q/p$, $\alpha_2=p^{-2}q(q+1)$, $\alpha_3=p^{-3}(q+1)(q+2)$: [2]55.

$E(x)=a(b+1)$, $v=a^2(b+1)$

r^{th} cumulant $=q(r-1)!\ p^{-r}$ [2]67.

$\mu_2=\sigma^2=qp^{-2}$, $\mu_3=2qp^{-3}$, $\mu_4=3q(q+2)p^{-4}$: [2]433.

Arithmetic, geometric means: Math. Student 13:11.

C(x): [c]25:379.

D(x/(x+y))=Beta: [14]41.

$\sim C(x)$: MR13:553.

Transformations $y=x^n$, $y=e^x$: [d]9:176.

Normalizing transform: Proc. Pak. Stat. Assoc. 5:120.

Transformation $y=(x+k)^{\frac{1}{2}}$: [d]14:115.

Moments, Ch(x), cumulants when $x=y-c$: [18]1-136, 1-144.

Type III (p, p+1): [c]5:176.

$D(\bar{x})$=Type III (np, nq): [2]244, [c]18:335, [w]1:73.

$D[(nq-1)n^{-1}\ \bar{x}^{-1}]$=Reciprocal Type III(p,nq−1):

$\sim D(\sqrt[3]{\bar{x}})$: [c]35:297.

$D(x/y)$=Beta of second kind ($p=q$): [w]*1:74*.

$FD(p^{-1})$=Type V ($n\bar{x}/nq-1$, $nq-1$): [3]*87*, [c] *30:408*.

Bayes $D(p^{-1})$=Rectangular: [3]*91*.

D(xy) where x is Type III and y is Type V: [i]*39:64*.

D(x−y)=No. 8.92, where D(x)=Type III (λ, N) and D(y)=Type III (μ, 1)

D(HM): *MR4:164*.

D(xy), Ch(xy): *MR16:377*.

D(xy)=Bessel, D(xyz): *Studies Presented to Richard von Mises*, p. 301, Acad. Press, 1954.

MLE (p)=Moment estimate (p)=$q/\bar{x}$, correcting for bias=$(nq-1)/n\bar{x}$, sufficient, ∼efficient, not efficient: [3]*26*.

$\mathrm{Var}((nq-1)/n\bar{x})=p^2(nq-2)^{-1}$: [1]*505*, [u]*45:214*.

$\mathrm{MLE}(1/p)=\bar{x}/q$: [3]21.

$\mathrm{Var}(\bar{x}/q)=p^{-2}n^{-1}q^{-1}$, sufficient: [3]*21*.

Sufficient statistics for p: [e]*17:212,219*.

Ordered LSE is MLE for 1/p: [d]*25:315*.

MLE (p,q), variance-covariance of estimates: [b] *14:187*, [c]*42:22*, [r]*1:18*.

$\mathrm{UMVUE}(1/p)=\bar{x}/q$, with $\mathrm{Var}=p^{-2}n^{-1}q^{-1}$: [3]*53*.

There is a sufficient estimate of q: [3]*26*.

Estimation: [e]*8:324*.

Minimax: [16]*64*, [e]*14:57*.

Gauging: [e]*15:192*.

Closest estimate: [u]*33:217*.

Testing n such populations: [3]*325*.

Slippage tests for p: *Koninkl. Nederl. Akad.* (A) *59:329*.

Testing equality of 1/p: [c]*31:205*.

Confidence intervals for 1/p: [3]*74*, [e]*6:113*, *MR5:128*.

Truncated distribution: [g]*45:411*.

Estimation from truncated Type III: [d]*26:659*, [d]*27:498*.

Truncated samples: [c]*40:52*.

D (gap between events)=generalized Poisson: [o] *3:123*, [o]*11:101*.

Characterized by independence of sum and quotient: [d]*26:319*.

Discrete Type III: [c]*44:365*.

Mills ratio: [d]*24:309*.

Normal limit: *Am. Math. Monthly 50:98*.

Renewal theory: [d]*11:448*.

Trivariate: [d]*21:550*.

See also: [d]*7:95*, [d]*8:17*, [d]*24:407*, [e]*2:150*, [b]*11:101*, [c]*21:263*, [d]*25:640*, [c]*24:300*, [v]*2:330*, [g]*50:904*, [i]*39:171*, [q]*7:95*, *Am. Math. Monthly 50:98*, *MR17:756*.

2.2. Type III (p, 1)

$D(x)=pe^{-px}$, "negative exponential" [5]*34*.

Type X.

$\alpha_1=1/p$: [5]*59*, [2]*48*.

$v=p^{-2}$: [5]*67*.

Moments: [2]*142*, [8]*100*, [10]*18*.

Characterization: [t]*7:60*, *3d Berkeley Symp. 2: 195*.

Cumulants: [2]*87*, [10]*40*.

C(x): [c]*25:379*.

Mean difference: [c]*28:432*.

$MGF(x)=(1-t/p)^{-1}$: [10]*37*.

MGF(log x): [r]*7:296*.

Grouping corrections: [c]*39:433*.

A priori distributions of p: [i]*27:36*.

D(x+y): [8]*95*.

If $D(x)=pe^{-px}$, $D(y)=Pe^{-Py}$,

$$\text{then } D(x-y)=\begin{cases} Ae^{-p(x-y)}, & x>y \\ Ae^{P(x-y)}, & x<y \end{cases}$$

where $A(p+P)=pP$:

Examples and applications: [8]*29,79,83*, [j]*20:366*, [c]*39:168*.

$D(\bar{x})$: [c]*39:168*.

$D(\xi)$=No. 8.9, $MGF(\xi)$: [p]*7:153*.

Σx_i where x_i is Type III(p_i, 1): *MR5:42*.

Rank variates: [c]*24:210,271*, [r]*4:153*.

Estimation: [c]*35:187*, [g]*48:493*, [s]*10:167*, [o]*8: 15*, [p]*7:152*.

Censored sample: [c]*41:230*, [d]*23:237*.

Moments of D(s): [c]*22:53*.

Variance of mean deviation$\simeq 4/3np^2$: [2]*217*.

Testing $p=p_0$: [d]*9:84*.

Sequential test: [c]*41:252*, [d]*27:460*.

Testing against four other possible dist: [c]*43:253*.

Confidence intervals: [3]*84*.

Estimation from truncated exponential: [d]*26:498*

Relation with Poisson, D. L. Gerlough and A. Schuhl, *Poisson and Traffic*, The ENO Foundation, 1955: [o]*2:13*.

See also: [d]*7:19*, [d]*25:555*, [g]*48:488*, [g]*50:904*.

If $x=y-c$, Mean$=c+\frac{1}{p}$, var$=\frac{1}{p^2}$, skewness=2, kurtosis=6, Ch(x), cumulants: [18]*1–136, 1–144*.

MLE: [k]*8:52*.

LR test for hypothesis that n such populations are identical, etc: *[3]305.*

k samples: *Z14:269.*

Best linear estimates of m and σ: *[d]25:320.*

$$D\left\{\frac{1}{n-1}\sum_{i=1}^{n-1}|x_{i+1}-x_i|\right\}: \quad [c]6:133.$$

Life testing: *[d]25:373.*

Original Neyman-Pearson paper on hypothesis testing: *[c]20:221.*

Censored samples: *[g]52:58.*

Estimation by order statistics: *[d]26:585.*

Quasi-range: *[d]28:179.*

LR tests: *[d]12:301.*

Seq. testing: *[x]2:86.*

Confidence intervals: *MR5:43.*

$$\left\{\begin{array}{l} H:\ p=p_0,\ c=c_0 \\ Alt:\ p=p_1,\ c=c_1 \end{array}\right\} \quad [3]304.$$

See also: *[d]25:409, [d]24:458, [c]30:402,416, [c]24:279.*

Maxwell-Bolzmann.

x^2 is Type III (p, 3/2): *[5]39,60, [s]1:304.*

Connection with N(0, v): *[12]40.*

$D(\bar{x})$: *[n]10–3:90.*

See also: *[n]17–1:125.*

2.3. Type III (1, q)

$D(x)=\frac{1}{\Gamma(q)}e^{-x}x^{q-1}$, "Gamma": *[10]149.*

$\alpha_r=\Gamma(q+r)/\Gamma(q)$: *[10]150,163.*

$k_r=q(r-1)!$: *[2]96,153.*

$Ch(x)=(1-it)^{-q}$: *[17]No. 34.*

Skewness $=q^{-\frac{1}{2}}$: *[10]161.*

HM $=q-1$: *[10]163.*

$D(n\bar{x})$ = Type III (1,nq): *[c]19:228, [n]10–3:91.*

$D(\Pi x_i)$: *[c]24:474.*

$D(x_1-x_2)$: *[2]252, [c]24:293.*

$D(x_1+x_2)$ = Type III $(1,q_1+q_2)$: *[10]151, [p]7:101.*

D(GM) as a series: *[2]251.*

With generalization: *[d]5:277.*

$D[x_1/(x_1+x_2)]=B(\frac{1}{2}q_1, \frac{1}{2}q_2)$: *[10]153.*

$D(x_1/x_2)$ = Beta of second kind: *[10]158, 160, [p]7:102.*

$D(x_1-x_2)$ involves a Bessel function if x_1 and x_2 are from two separate distributions and $D(x_1/x_2)$ is Fisher: *[d]7:51.*

$\sim D(\bar{x})$: *[d]25:636.*

$D(\bar{x}, GM/\bar{x})=D(\bar{x})\ D(GM/\bar{x})$: *[c]30:287.*

C.—R.$(q)=[n(d^2/dq^2)\log\Gamma(q)]^{-1}$

$\bar{x}$ is moments estimate of q, not sufficient.

Closest estimate: *[u]33:216.*

MLE(q) = log GM: *[p]7:169.*

$E(\bar{x})=q$, var $(\bar{x})=q/n$, efficiency $(\bar{x})\to 0$: *[1]504,5.*

$\sim D(\log GM)$ = Normal: *[1]505.*

Confidence intervals: *[p]7:224.*

Mellin transform: *[d]19:373.*

Log log (1/x): *[v]8:71.*

A + B log x: *[c]36:165.*

Multivariate generalization: *[d]22:549.*

Tetrachoric functions: *[c]14:161.*

Fermi-Dirac, x—c is Type III (3/2,const.): *[12]42, [j]8:701, [s]1:304.*

See also: *[d]25:401, [10]161, [d]22:425, [c]24:281, [c]27:409, [c]30:415, [c]36:165, [d]25:784, [e]10:314, [g]51:467, [c]44:265, [d]22:418, Can. J. Math. 3:140.*

2.4. Type III (1, 1)

$D(x)=e^{-x}$, "exponential": *[6]217.*

$Ch(x)=1/(1-it)$: *[17]No. 33.*

$D(\bar{x},s)$: *[d]3:128, [d]4:133, [d]4:139,142.*

x = y − c, Type X, confidence intervals: *[b]17:90.*

$D(\Sigma x_i)$ = Type III by convolutions: *[d]5:13.*

$D(\Sigma x_i/i)=D(\max x_i)$: *[b]14:43.*

Doubly truncated, Ch(x): *[n]10:3, [17]No. 32.*

Ratio of two ranges: *[d]21:112.*

C.—R. theorem false for x = y − c: *[1]485, [3]47.*

See also: *[d]22:425, [i]36:152, [m]6:120, [d]22:418.*

2.5. Chi-Square (k)

$D(x)=[1/2^{\frac{1}{2}k}\Gamma(\frac{1}{2}k)]x^{\frac{1}{2}k-1}e^{-\frac{1}{2}x}$: *[5]96, [10]164, [2]17, [4]102, [8]134, MR8:161, [1]3:353, [18]1—161.*

Type III$(\frac{1}{2}, \frac{1}{2}k)$:

$\alpha_1=k$, $v=2k$: *[1]234, [4]103.*

$\alpha_s=k(k+2)\ \ldots\ (k+2s-2)$: *[1]234.*

$\mu_2=2k$, $\mu_3=8k$, $\mu_4=48k+12k^2$, $\mu_5=32k(5k+12)$: *[2]292.*

Cumulants: *[c]31:216.*

$Ch(x)=(1-2it)^{-\frac{1}{2}k}$: *[e]4:8.*

C(x), relation with Poisson: *[l]3:357, [c]37:313.*

Introduction, properties, examples: [15]253.

Obtained as dist. of normal variance: [3]104, Z23: 148, [p]7:98.

$D(x_1+x_2)$: [e]7:27.

D(log x): [c]34:170.

$D(2\sqrt{xy})$=Chi square(2n−2) if D(x)=chi-square(n) and D(y)=chi-square (n−1).

$D(\bar{x},s)$ for k=2,3,4, n=3,4: MR12:345.

$D(x_1/x_2)$=Beta of second kind $(\frac{1}{2}k_1, \frac{1}{2}k_2)$: [10]177.

$D[x_1/(x_1+x_2)]$=Beta (k_1, k_2): [10]177.

Obtained as $D(\Sigma x_i^2)$: [10]169.

$\sim D[(x-k)(2k)^{-\frac{1}{2}}]=N[k,(2k)^{-\frac{1}{2}}]$, $\sim D(2x)^{\frac{1}{2}}=N[(2k)^{\frac{1}{2}}, 1]$: [1]251.

$\sim D(-2 \log LR)$: [d]9:60.

Reproductive property: [4]105, [10]177.

Normal approximation: [d]17:216, [d]27:786.

Large parameter: [c]43:92, Proc. A.M.S. 6th Symp. in Appl. Math., p. 251.

Convolution for a pair of Chi-square variables: [8] 134.

Percentage points: [c]41:313, [i]33:168.

∼Significance levels: [d]14:57,93.

Elderton's tables: [c]1:155.

Minimax estimation: [16]17.

Connection between χ^2 test and χ^2 distribution: [c]19:215.

Original Neyman-Pearson paper on hypothesis testing: [c]20:263.

Queueing: [b]16:82

Approximation for small sample: [d]9:158.

See also: [d]18:89, [g]29:372, [c]22:298, [a]85: 87,95, [a]87:442, [c]29:133, [c]29:389, [c]31:346, [c]34: 368, [c]32:268, [c]40:421, [p]7:98.

2.6. Non-Central Chi-Square

$$D(x)=e^{-\frac{1}{2}x}e^{-\frac{1}{2}k}2^{-\frac{1}{2}n}\sum_{j=0}^{\infty}\frac{x^{\frac{1}{2}n+j-1}k^j}{\Gamma(\frac{1}{2}n+j)2^{2j}j!}$$: [c]36:204.

$$Ch(x)=\frac{\exp[kit/(1-2it)]}{(1-2it)^{n/2}},$$

$k_r=2^{r-1}(r-1)!(n+rk)$, ∼form: [18]1–162.

Logarithmic noncentral Chi-square: [o]7:57.

See also: [i]34:57, [c]41:538, [i]36 (supplement): 18, [d]28:678 [c]44:528.

2.7. Helmert (p, q)

$$D(x)=\frac{(x/q)^{p-1}e^{-\frac{1}{2}x^2/q2}}{q\Gamma(\frac{1}{2}p)2^{\frac{1}{2}(p-2)}}$$: [5]94

For $2q^2=k$, p=2, called Rayleigh: [12]39.

$D(x^2/q^2)$=Chi-square:

Called seminormal: [i]20:61.

Refs, Ch(x): [17]No. 42

$$\alpha_1=\frac{[\frac{1}{2}(p-1)]!}{[\frac{1}{2}(p-2)]!}2^{\frac{1}{2}}q, v=pq^2-\alpha_1^2$$

Non-central: [d]23:467.

See also: [c]23:418, Elec. Eng., Nov. 1954, p. 1004, MR8:161, J. Appl. Phys. 23:137.

2.8. Reciprocal Type III (p, q)

$$D(x)=\frac{p^{q+1}q^{q+1}}{\Gamma(q+1)}x^{-q-2}e^{-pq/x}, pq>0,$$

$$\text{Mean}=p, \text{ var }=\frac{p^2}{q-1}, \text{ Type V},$$

$\alpha_r=(pq)^r\Gamma(q-r+1)/\Gamma(q+1)$: [2]86, 142.

Obtained from Pearson equation: [11]45.

Various constants, with an example: [11]78, [d]3:20.

Ch(x) in special case: [17]No. 47.

If $q=-\frac{1}{2}$, $p=-1$, $Ch(x)=\exp\ i(-1+i)\ \sqrt{t}]$: [s]208:318, [17]No. 49.

More generally: [d]7:25.

MLE: [r]1:19.

As distribution of precision constant in normal sample: [a]97:132.

C(x): [c]25:379.

See also: [c]26:388, [c]36:165, [o]8:55.

2.9. Generalized Type III

$D(x)=C(1+x/a)^{ps}e^{-px}$, with moments of D(s): [c]22:52.

$D(x)=[p^pe^{-p}/q^p\Gamma(p)](q+x)^{p-1}e^{-px/q}$, with semi-invariants, $D(\bar{x})$ete: [c]21:287, [c]24:293.

$Ch(x)=e^{-a|x}[1-(ix/p)]^{-ap-1}$: [17]No. 35.

$D(x)=[1/p\Gamma(q)][(x-c)/p]^{q-1}\exp[(c-x)/p], c\leq x<\infty$.

MLE(c,p,q): [3]39.

Variance of estimates: [3]42.

Tables: [d]1:191.

Estimation: [g]48:336.

$D(x)=[e^{-bx}b^p/\Gamma(p)]e^{-bx}(x+a)^{p-1}$ from Pearson's equation: [4]74, [2]124, [11]65,

$D(\bar{x})$: [n]10–3:91.

$D(x)=A(x-c)^{q-1}e^{-p(x-c)}$, $x>c$, $p>0$, $q>0$· [1]249.

Various constants, with an example: *[11]66.*

One root of quadratic in Pearson equation is ∞: *[11]44.*

$D(x_1/x_2)$ where each is Generalized Type III, or one is Generalized Type III and the other $N(m,v)$: *[2]253.*

Bayes' Theorem: *[n]16–1:114.*

Counting radioactive particles: *[d]18:260.*

Two Generalized Type III distributions added: *[n]8–3:76.*

See also: *[d]1:150, [d]1:191, [d]7:18, [c]1:293, [c]3:311, [c]5:173, [c]13:13, [c]16:114, [n]1–3:88, [c]32:294, [u]46:284.*

2.10. Wishart Univariate

$D(x) = \frac{a^{\frac{1}{2}(n-1)}}{\Gamma[\frac{1}{2}(n-1)]} x^{\frac{1}{2}(n-3)} e^{-ax}$, Type III$[a, \frac{1}{2}(n-1)]$: *[1]391.*

3. Binomial Distributions

3.1. Binomial (k, p)

$D(x) = \binom{k}{x} p^x (1-p)^{k-x}$, $x = 0,1, \ldots, k$, "Bernoulli": *[4]47, [10]46, [5]106, [d]1:118.*

$\alpha_1 = kp$, $v = kpq$, (where $p+q=1$), $\alpha_3 = kpq(q-p)$, $\alpha_4 = 2k^2p^2q^2 + pq(1-6pq)$: *[1]193, [c]5:172, [5]57,66, [10]58, [2]52, 117.*

$\mu_{r+1} = pq(kr\mu_{r-1} + d\mu_r/dp)$: *[2]118*

$\beta_1 = (q-p)/\sqrt{kpq}$, $\beta_2 = 3 + (1-6pq)/kpq$.

Factorial moments $\alpha_{[1]} = \kappa^{[1]}p^1$, where $k^{[1]} = k(k-1) \ldots (k-i+1)$: *[1]257, Nature 164:282, [2]87.*

Long introductory article: *[15]23.*

Moments: *[d]6:96, [c]3:325, Z9:28.*

Cumulants $\kappa_{r+1} = pq(d\kappa_r/dp)$: *[2]135, [c]31:392 [18]1–144, [d]2:196.*

Mean deviation: *[c]44:532, MR7:128.*

Several formulas for moments about the mean: *[d]7:191, [c]15:410, Z9:220.*

$C(x)$: *[c]38:423.*

$C'(x)$ as a Beta integral: *[6]235, [18]1–152.*
Recursion formula for moments: *[14]33.*

Moments in general: *[d]8:103, [d]11:106, [c]17:165 [c]26:262, MR7:461, [c]30:11, Bull. Am. Math. Soc. 40:262, 41:857.*

Moments and series: *[i]14:168.*

Arcsin transform: *[14]210.*

Another form of $D(x)$: *[d]8:116.*

$\beta_1 = (q-p)^2/kpq$, $\beta_2 = [1+3pq(k-2)]/kpq$, etc.: *[12]52, [d]4:216.*

$kp - q \leq \text{mode} \leq kp + p$: *[6]57.*

$Ch(x) = (pe^{it} + 1 - p)^k$: *[5]62, [2]55,103.*

$MGF(x) = (q + pe^t)^k$: *[4]48, [10]38.*

$PGF = (q+pt)^k$: *[18]1–146.*

$\sim C(x)$: *[j]8:99.*

$E(1/x)$: *[g]49:169.*

$D(n\bar{x}) = \text{Binomial } (nk,p)$: *[2]243.*

$D(s^2)$: *[c]44:262.*

$FD(p)$: *[c]37:117.*

$D(x-y)$ in terms of Legendre functions: *MR14:566.*

Reproductive property by convolutions: *[7]216.*

A convolution with respect to p: *[w]6:165.*

$C.-R.(p) = pq/kn$: *[5]141, [15]207, [p]7:160.*

$\bar{x}$ is sufficient: *[p]7:162.*

$(kn)^{-1}\Sigma x_i$ is efficient and unbiased: *[5]141, [1]487.*
Is MLE: *[5]144.*

$\sim$BCR for $k = k_0$: *[d]18:556.*

Minimax and Bayes: *[d]23:404.*

Minimax estimation: *[16]18, [d]21:190.*

Modified Bayes: *MR11:42.*

BANE(p): *[d]21:402.*

Biased and unbiased statistics: *[t]5:149.*

MLE(k,p): *[k]18:117.*

Estimation of p based on runs: *MR14:1102.*

LR comparison of two binomials: *[c]37:140.*

Confidence intervals: *[w]2:171, [c]41:275, [c]33:181, [w]5:94, [d]9:174, [y]21:17, [4]129, [w]19:130, [w]2:171, [c]41:308.*
Tables: *[3]81, [c]44:436, [o]8:85.*

Sequential: *[d]17:288, 489, [d]18:131, [b]8:98, [c]41:252.*
Acceptance inspection: *[b]12:301, MR15:727.*

Chi-square test: *[k]7:207.*

UMP test: *[c]43:465.*

Sampling inspection: *[j]8:626.*

Multiple sampling: *[d]14:363.*

Analysis of variance: *[d]11:335.*

Order statistics: *MR16:729, [s]8:62.*

Approximate formulas: *[8]172, [d]19:592, [n]18 No. 1–2:123.*

Normal approximation: *[9]131, [d]16:319, [c]4:190, [c]29:402, [r]4:47, Proc. Koninkl. Nederl. Akad. (A)57:513, MR10:131.*
Using generating functions: *Proc. 5th Intl. Cong. Math 2:441, [w]1:41.*

Inequalities for tails: *[7]126.*

Asymptotic behavior: *MR15:138.*

Convergent sequences of binomials: *Am. Math. Monthly, 50:96.*

n binomials: [e]*8:11.*

Normalizing transform $y=k^{\frac{1}{2}}\sin^{-1}(x+a/k)$ and other transforms: [d]*14:116*, [f]*3:52*, [c]*35:248.*

$\log x/(1-x)$, $-2\tanh^{-1}x$: [r]*8:73.*

Other transforms: [x]*2:94.*

Choosing between several binomials: [j]*36:537.*

If p not constant (cf. "Lexian"): [k]*16:1.*

Transformations, approximations, applications: [15]*668.*

Chain binomials: [c]*40:279.*

Gambler's ruin: [i]*24:52.*

Connection with Beta distribution: [c]*41:304*, [n]*18:121.*

Actuarial application: [i]*35:11.*

Binomials added: *MR17:862*, [w]*5:73.*

Generalization: [d]*20:311.*

Generalized binomial $Ch(x)=\Pi(q_i+p_ie^{itx})$: [2]*122.*

A modification: [e]*15:237, 251.*

See also: [d]*6:27*, [d]*17:13*, [d]*21:247*, [d]*22:129*, [e]*12:248*, [b]*14:115*, [j]*8:364*, [c]*11:269*, [c]*16:165, 202*, [f]*12:276*, [f]*13:225*, [r]*1:15, 32*, *Bull. Am. Math. Soc. 1935, p. 857*, *11th Skand. Math-Kongress p. 210*, *Z5:212*, *Z18:31*, *MR6:234*, *MR11:604*, *MR12:509*, *MR9:450*, *Z3:18*, [w]*1:9*, [g]*33:390*, [a]*83:277*, [c]*33:222*, [g]*49:169*, [i]*7:153*, [i]*6:77*, [i]*20:77*, [i]*22:23*, [i]*26:22*, [i]*31:8*, [i]*32:188*, [d]*25:770*, [c]*44:364*, *J. Proc. Roy. Soc. N.S.W. 81:38*, *Z19:316*, *Proc. Intl. Cong. Math. (1924) 2:801*, [y]*3:282*, [w]*8:23.*

3.2. Binomial (1, p)

$D(x)=p^x(1-p)^{1-x}$, $x=0,1$.

$D(n\bar{x})=$ Binomial (n,p): [6]*207*, [w]*1:73.*

Chi-square test: [e]*13:3.*

Confidence intervals for p: [6]*233.*

Completeness: [e]*10:315.*

UMVUE $(p)=\bar{x}$, UMVUE $(pq)=n\bar{x}(1-\bar{x})/(n-1)$.

See also: [v]*3:324*, [w]*1:9.*

3.3 Truncated Binomial (k, p)

$D(x)=\binom{k}{x}(p^xq^{k-x}/(1-q^k))$, $x=1,\ldots,k$: [6]*162.*

$E(x)=kp/(1-q^k)$, $v=kpq/(1-q^k)$, moments of x^{-p}: [d]*16:50.*

Estimation: [g]*50:877.*

Tables: [g]*49:169*,
With an application: [k]*14:321.*

See also: [b]*11:2*, *MR15:969.*

3.4. Negative Binomial (*k*, *p*)

D(x)	p	q=1−p	Mean	Variance	Reference
$\binom{-k}{x}p^x(1-p)^{-k-x}$	p	1−p	−kp	−kpq	
$\left(\frac{a}{1+a}\right)^k\binom{-k}{x}\frac{(-1)^x}{(1+a)^x}$	−1/a	$\frac{a+1}{a}$	k/a	k/a+k/a²	[1]*259*, [a]*83:255*
coefficient of t^x in $\left(\frac{a}{1+a}\right)^k\left(1-\frac{t}{1+a}\right)^{-k}$	−1/a	$\frac{a+1}{a}$	k/a	k/a+k/a²	[2]*125*
$p^k\binom{x+k-1}{k-1}(1-p)^x$	$\frac{p-1}{p}$	1/p	qk/p	qk/p²	[d]*17:53*, [6]*61*, [18]*1–158*, [17] *No. 6*, [7]*218*
$\binom{x+k-1}{k-1}\frac{p^x}{(1+p)^{k+x}}$	−p	1+p	p	p+p²	[f]*9:176*, [4]*54*
$\left(\frac{m}{1+bm}\right)^x(1+bm)^{-1/b}\frac{1}{x!}\prod_{j=1}^{x-1}(1+jb)$	−bm	1+bm	m	m(1+bm)	[5]*32*
$\left(\frac{n}{n+km}\right)^n\binom{x+n-1}{n-1}\left(\frac{km}{n+km}\right)^x$	−km/n	$\frac{n+km}{n}$	km	$km+\frac{k^2m^2}{n}$	[c]*41:78*

If $Qp=1$, $Q(1-p)=P$, then $\beta_1=(P+Q)/\sqrt{kPQ}$, $\beta_2=3+(1+6PQ)/kPQ$.

Obtained by assuming a Poisson parameter to be Type III: [1]259, [2]125, [c]41:78, [a]110:132, [f]5:162.

Some derivations, with interesting properties: [18]1–159, [c]44:530.

If $k=h/p$, called Polya-Eggenburger: [4]55.

$Ch(x)$: [17]No. 7, extension, *Mem. Fac. Sci. Kyushu Imp. Univ. (Ser A) 1:178.*

A "contagious" distribution: [7]83, 101, [13]413, [7]128.

Skewness, kurtosis, cumulants: [18]1–136, 1–144.

$C(x)=(p+q)^{-n}$: [c]37:209.

$Ch(x)=[1+bm(1-e^{it})]^{-1/b}$: [5]62.

Called compound Poisson: [15]727.

$Ch(x)$: [v]4:9.

Limit of contagious: [c]41:269.

Limited by Poisson and Pascal: [7]233.

PGF: [18]1–146.

Problem leading to Negative Binomial, with generalization: [d]17:53.

Generalization: *MR16:602.*

Paper by Fisher: [k]11:182.

Recurrence formula for cumulants: *Aktuárské Vědy 5:182.*

Moments: *G. Dell Ist. Ital. degli Attuari 6:3, Z13:70.*

Formulas for tails: [7]237.

MLE: [b]8:206, [c]37:114.

$M\chi^2E$: [k]11:109.

UMVUE(p): [c]18:374.

Estimation: [d]24:409, *Psychometrika 16:107.*

Sequential: [f]6:59.

Sampling: [c]37:358.

Fitting: [f]9:176.

Truncated: [g]50:877.

Moments, estimation, MLE: [c]42:58.

Bhattacharyya bounds: [d]27:1182.

Transformation $\sinh^{-1}x$: [f]3:52, [c]35:249.

Transformations: [c]41:315.

Called "Pascal," satisfies:
$D(x+1)=[(1-p)(x+k)/(x+1)]D(x)$, etc.: [i]14:176.

Accident proneness: [c]37:24.

Telephone traffic: [j]35:454.

Bibliography: [1]437.

See also: [7]236, [b]10:260, [f]5:165, [f]7:340, 411, [c]35:11, [c]39:178, 198, [c]40:203, [c]40:370, [c]44:364, [i]20:78, [i]22:25, [i]31:9, *J. D'Analyse Math 1:331, Psych. Bull., 47:434,* [u]45:364, *Z6:69, Z18:265. Z13:409, Z14:29, MR17:944,* [a]99:733, [w]8:23.

3.5. Negative Binomial (1, −m)

$D(x)=[1/(1+m)][m/(1+m)]^x$, "Pascal," or "Furry," or "Geometric" $\alpha_1=m$, $v=m^2+m$,
$Ch(x)=[1+m(1-e^{it})]^{-1}$: [5]31,60,66.

Cumulants: [18]1–144.

PGF: [18]1–148.

If $D\left(\frac{m}{1+m}\right)$=Beta, then D(x)=No. 8.93

See also: [c]39:346, [7]59, [v]4:8, [c]36:165, [15]38, [c]44:265.

3.6. Discrete Lexian

$D(x)=\Sigma f(p)\binom{k}{x}p^x(1-p)^{k-x}$, moments, etc.: [i]26:34.

"Generalized Binomial".

Poisson-Lexian: [i]26:57.

$Ch(x)=[p\phi(t)+q]^k$: [17] No. 69.

If *a priori* distribution of p is Beta: [i]27:39, *Bull. Am. Math. Soc. 41:860.*

3.7. Deterministic

$D(x)=\left\{\begin{matrix}0,\ x\neq c\\ 1,\ x=c\end{matrix}\right\}$: "casual" [1]192.

$Ch(x)=e^{ict}$: [8]209, [5]29, 62.

For c=1: [2]96.

Bibliography: [17] No. 1.

Moments, cumulants, Ch(x), PGF: [18]1–136, 1–144, 1–146, [v]3:324.
See also: [c]44:366, *Z9:363,* [w]1:9.

3.8. Reciprocal Truncated Binomial

D(x): [n]18 No. 1–2:77.

4. Discrete Distributions

4.1. Poisson (m)

$D(x)=e^{-m}m^x/x!$, x=0, 1, . . . , law of small numbers: [6]59, [5]30, [8]X, [7]72, 115, [10]47, 63, [15]119.

$Ch(x)=\exp[m(e^{it}-1)]$: [1]204, [5]62, [2]66.

$MGF(x)=e^{-m}\exp(me^t)$: [6]101, [4]53, [m]2:46.

$\alpha_1=m$, $\alpha_2=m^2+m$, $v=m$: [6]102, [5]57, 66, [4]53.

$\alpha_3=m[(m+1)^2+m]$,
$\alpha_4=m(m^3+6m^2+7m+1)$: [10]59.

$\mu_2=m$, $\mu_3=m$, $\mu_4=m(1+3m)$, $\mu_5=m(1+10m)$, $\mu_6=m(1+25+15m^2)$: [d]1:119, [2]86.

Skewness, kurtosis: [18]1–136.

PGF: [18]1–146.

$\mu_{r+1}=rm\mu_{r-1}+m(d\mu_r/dm)$: [2]121, *Bull. Am. Math. Soc. 40:264, 41:857.*

All cumulants=m: [2]66.

Factorial moments $\alpha_{[1]}=m^1$: [1]257.

Moments in general: [d]8:103, [i]14:173.

Recursion formula for moments, correction with multinomial, C(x) as a Γ integral: [14]36–8.

$\beta_1=1/m$, $\beta_2=3+1/m$, $\gamma=1/m$: [12]52.

C(x): [j]5:604, *MR4:194*, [c]37:313.

Transform $y=\sqrt{x}$: [14]209.

$D(\bar{x})=e^{-nm}(nm)^{n\bar{x}}/(n\bar{x})!$: [1]379, [2]243, [6]208, [15]219.

D(x+y): [10]59.

$\sim D\left(\frac{x-m}{m}\right)=N(m, m^{\frac{1}{2}})$: [1]250.

D(x−y): [2]251, *MR14:566*, [a]109:296, [a]100:415, [e]7:175.

∼D(x−y): *MR15:138.*

D(gap between two Poisson events)=exponential: [c]41:252, [g]49:255, *MR14:293*, *Am. Math. Monthly 64:719.*

$D[\bar{x}^{-1}\Sigma(x_i-\bar{x})]$, i.e., Chi-square test: [b]5:75, *Z18:321.*

D(Index of dispersion): [d]14:155, [c]40:225.

Various *a priori* distributions of m, in particular Type III: [i]27:33.

$E(x^2)=E(x+1)$: [8]119.

E(1/x): [g]49:169.

Var $(x^{\frac{1}{2}})$: [a]106:143.

Reproductive property by convolutions: [7]216. by Ch. functions: [9]279.

C.−R. (m)=m/n: [1]487.

C.−R. $(m^2)=4m^3n^{-1}$.

BANE: [d]21:401.

Estimation when m must be integral: [b]12:213

Estimation: [d]24:406.

Estimation from censored samples: [g]49:158.

Estimation of bacteria population: [c]31:170.

Estimation of m or 1/m: [g]49:255.

Mx^2E: [3]56.

Approximation, estimation, application: [15]714.

MLE(m)=$\bar{x}$: [4]141, [9]6.22, [5]144, [3]21, [p]7:169.

$\bar{x}$ is sufficient: [4]136, [w]22:713.

$\bar{x}$ is efficient: [1]487.

$\bar{x}$ is unbiased: [3]142.

Completeness: [e]10:315.

Confidence intervals, tail: [c]41:312.

C.–R: [16]16.

Confidence intervals: [3]71, 81, [d]9:173, [c]28:437, [c]44:436, [e]14:25, [p]7:223.

Order statistics: *MR16:729.*

Approximate moments of ordered variables: [s]8:78.

Testing whether two Poissons are the same: [3]127, [c]37:143.

Two Poissons, etc.: [c]40:447.

Whether k Poissons are the same: [d]16:362, *Proc. Nat. Inst. Sci. India 3:297.*

Analysis of variance: [d]11:335, *J. Econ. Entom. 37:717.*

Testing m: [c]31:314, [c]40:354, [g]49:255, [14]205.

Testing ratios of means: [o]4:45.

Chi-square test: [k]7:207.

Testing against contagious: [c]37:59.

Sequential testing m: [d]19:400.

Small sample tests: [f]12:264.

Monograph on Poisson testing and estimation: *MR16:383.*

Obtained from a difference equation analogous to Pearson's differential equation: [f]2:419.

Obtained from postulates: *Z13:408.*

Early discussion with numerical examples: [c]10:36.

Transform $y=(x+k)^{\frac{1}{2}}$: [d]14:113, [f]3:52, [c]35:247.

Transformations: [c]41:312.

Domain of attraction: *MR3:2.*

As limit of binomial: *MR12:190*, [4]52, [7]110, [i]6:78, [s]1:161.

Generalizations: [c]36:18, *Operations Res. 3:198*, [c]37:48.

A modification: [e]15:237, 251.

Convergent sequence of Poissons: *Am. Math. Monthly 50:97.*

Connection with hypergeometric: [c]25:300.

If m Poisson, called double Poisson: *Kendall and Buckland, A Dictionary of Stat. Terms.*

Normal approximation: [7]146, *MR16:1034*, *MR10:613*, [r]4:37.

Compounded with binomial: [7]128,221.

As approximation to Beta: [j]20:19.

An approximation: *MR18:423.*

Connection with Gram-Charlier: [2]154.

Connection with Type X: [o]2:13, *D. L. Gerlough and A. Schuhl, Poisson and Traffic, The ENO Foundation, 1955.*

Connection with Type III: [o]3:123.

Limiting theorems: [8]148.

Poisson if and only if $\sum_{i}^{a} xD(x) = m\,C(a-1)$

Characterization: [u]48:206, *Proc. A.M.S. 1:813, C. R. Acad. Sci. Paris 239:1114, 3rd Berkeley Symp 2:145.*

Generalizations: [d]13:410, [d]14:394, *MR15:138, MR16:1034,* [d]19:414, *MR13:258.*

Possibility of a continuous analog: [i]14:43.

Traffic control: [b]7:65.

Accident causation: [b]7:89, [a]90:487.

Poisson as a limiting distribution in five different ways, relation with multinomial, exponential: [18]1–156.

Accident proneness: [c]37:24.

Pedestrian delay: [c]38:383.

Insurance risk: [i]40:72.

Frequency of war: [a]107:242, [a]112:446.

Nomograph for acceptance inspection: [s]4:204.

Telephone switchboards: [5]30, [j]6:468.

An early treatment, with the famous example of the Prussian horse-kicks: *Von Bortkewitsch, L. Das Gesetz der Kleinen Zahlen, B. G. Teubner, Leipzig 1898.*

Other applications: [7]119, [j]5:604.

See also: [d]14:155, [d]20:523, [d]22:94, [d]22:128 [13]405, [j]7:45, [c]11:267, [c]11:211, [c]26:108, [g] 33:390, [g]42:574, [a]83:255, [f]6:17, *MR1:246, Z22:243, MR17:53, MR14:485, Z18:31, Z14:138, MR4:20, MR13:633, MR15:541, 634, MR7:310,* [f]7:340, [c]27:272, [c]30:188, [c]36:250, [n]16–2:285, [k]9:406, [c]38:427, [c]39:346, [i]20:80, [i]22:25, [i] 25:158, [i]26:46, [i]31:9, *Rio. Ital. di Demogr. e Stat 3:219, Proc. First Pakistan Stat. Conf. (1950) p. 59, Publ. Math. Debrecen 2:66, Annals of Applied Biology 9:325, Annales de l'Ecole Normalle Superiere 54:321,* [e]2:330, [e]3:327, [d]26:147, [c]44:265, 365, *Brit. Assoc. Math. Tables (3rd Ed.) V.1 p. xxxvi, Am. Math. Monthly 50:97, Annals of Math 37:357, Bull. Am. Math. Soc. 1935 p. 861, Am. J. Math 57:827,* [u] 45:219, *Kungl. Lantbruk. Ann. 18:86, Z2:200, MR1:15, MR18:341, MR5:128, MR2:112, Z15:407, Z18:412, MR14:1098, MR13:958,* [w]1:9, [s]1:93.

4.2. Truncated Poisson

$D(x) = m^x e^{-m}/x!(1-e^{-m})$, $x=1, 2, \ldots$: [f]8:275, [f]10:402, [f]11:387.

Estimation: [c]39:247, [c]40:171, [g]50:906, [i] 39:19, [f]9:485.

UMVUE: [c]18:374.

Tables: [g]49:169, [11]158, [13]39:247.

Servicing machines: [b]13:71.

D(1/x): [n]18 *No. 1–2:77.*

Doubly truncated, D(x) etc: [g]49:160, *Conn. Agric. Exp. Sta. Bull. No. 513.*

4.3. Compound Poisson

$D(x) = k_1(e^{-m}m^x/x!) + k_2(e^{-n}n^x/x!)$: double Poisson: [5]151.

If $k_1 = k_2 = \frac{1}{2}$: [5]150, [g]42:407, [f]8:281.

Compound Poisson $D(x) = (1/x!)\Sigma m_i^x e^{-m_i} k_i$: [7] 237, *MR17:862, R. D. Evans, The Atomic Nucleus, p. 766, MR13:633, MR14:770.*

4.4. Uniform

$D(x) = 1/k$, $x=1, \ldots, k$: [6]61.

Discrete rectangular.

MGF and moments: [d]11:324.

Sampling from: [c]21:126.

Estimation of range: [g]46:375.

Range and quotient of ranges: *Int. Congr. Math. (1950) 1:583.*

$D(n\bar{x})$: [p]10:52, 255, 324.

See also: *MR16:376.*

4.5. Hypergeometric

$D(x) = \left[\binom{m}{x}\binom{n}{r-x}\right] \Big/ \binom{m+n}{r}$: [6]61, [7]33, [15]40.

D(x) in another form, moments, etc: [2]126.

In the form of a hypergeometric series, asymptotic forms: [c]25:295, [c]26:59.

Various forms: *J. Soc. Stat. Paris 96:262.*

C(x) as a power series: [c]41:317.

Ch(x), refs: [17] *No. 8.*

$E(x) = mr/(m+n)$, $v = [mnr(m+n-r)]/[(m+n)^2 (m+n-1)]$: [7]183, [6]98.

Difference equation, moments, etc: [i]14:178.

Factorial moments: [2]135, *Nature 164:282,* [i] 6:79.

Skewness: [18]1–136.

PGF: [18]1–146.

Moments in general: [d]8:103, [d]10:198, [c] 16:157, [c]17:57, [c]26:264, [a]89:326, [e]3:326, *GANITA 7:1.*

Binomial and Poisson as limits: [15]690.

Binomial as limit: [7]47.

Poisson as limit: [7]114, [c]25:300.

Normal as limit: [7]146.

Normal, Poisson, binomial approximations: [18] 1–155, [d]27:471.

Minimax estimation: [d]21:191.

Completeness: [e]10:315.

"Confidence Limits for the Hypergeometric Distribution: *Chung and DeLury Univ. of Toronto Press, 1950, reviewed*, [a]115:286.

Generalization: [7]39, [c]41:266, *see also No. 8.59*, [b]18:202.

Satisfying difference equation [w]3:5

Linguistic application: [b]12:27.

Truncated hypergeometric, moments: [d]16:59.

Double hypergeometric: [7]187, *Koninkl. Nederl. Akad(A) 60:121.*

See also: [d]1:113, [d]21:248, [e]11:153, [j]7:39, [j]10:281, [f]8:287, [n]1–4:49, [c]37:140, [h]2:435, [i]22:26, [d]25:762, *J. Proc. Roy. Soc. N.S.W. 81:38, N.B.S. Math. Table MT19, MR17:984, Z12:29, Z13:273, MR1:340, U. Calif. Pub. Stat 1, No. 7, MR13:962, MR14:775*, [w]1:9.

4.6. Contagious

Obtained by considering $\int$(Poisson) dF(m): *MR14:293*, [d]14:389, [e]8:13.

F step function yields $D(x)=(1/x!)\Sigma p_i e^{-a_i} a_i^x$, (*No. 4.3*)

F Type III yields Polya-Eggenberger

$$D(x)=\frac{1}{x!}\,\frac{\Gamma(x+h/d)}{\Gamma(h/d)}\,(1+d)^{-d/h}(1+d)^{-x}.$$

m itself also Poisson yields $D(x)=e^{-k}(c^x/x!)\,\Sigma(i^x/i!)\,(e^{-c}k)^i$.

Neyman contagious Type A: [c]40:208.

Fitting: [f]11:149, *F. N. David, Probability Theory for Statistical Methods, p. 68, Cambridge 1949.*

Ch. fcns: [g]49:368.

Generalization: [f]9:354.

Testing against Poisson: [c]37:59.

Rutherford contagious: [d]25:703.

Σ_j (Poisson) $f_j(x)$, Ch(x), sp. cases: [17]*Nos. 66–8.*

Bivariate: *MR15:138.*

See also: [13]411, [f]9:354, [c]36:450, [c]39:346, [c]40:186, [c]41:268, [d]10:35, *J. Econ. Entom. 35: 536, MR1:251.*

4.7. Pollaczek-Geiringer

D(x), Ch(x), multiple occurrence of rare events: [17]*No. 9.*

4.8. Borel-Tanner

$D(x)=(e^{-\alpha x}\alpha^{x-r}x^{x-r-1}r)/(x-r)!$, $x=r, r+1, \ldots$, $0\le\alpha\le 1$, $r=1,2, \ldots$: *C.R. Acad. Sci., Paris 214:452*, [c]40:58, [c]47:143.

$m=r/(1-\alpha), v=[\alpha r/(1-\alpha)^3]$.

If $r=1$, MLE$(\alpha)=(\bar{x}-1)/\bar{x}$.

$PGF(x)=t^r \exp r\sum_{n=1}^{\infty}(n^{n-1}\alpha^n/n!e^{\alpha n})(t^n-1)$.

4.9. Polya

$$D(x)=\binom{N}{x}\frac{\prod_{i=0}^{x-1}(m+iR)\prod_{j=0}^{N-x-1}(n-m+jR)}{\prod_{k=0}^{N-1}(n+kR)}:$$ [7]128, [d]28:1021.

Contains Polya-Eggenburger No. 3.4, and Exceedance No. 4.10.

4.10. Exceedance

$$D(x)=\frac{\binom{n}{m}m\binom{N}{x}}{(n+N)\binom{N+n-1}{m+x-1}},\quad x=0, 1, \ldots, N:$$

[d]25:762, [w]6:164, [d]28:1021, [d]21:247.

Moments: [w]6:165.

4.11. Inverse Hypergeometric

$$D(x)=\frac{\binom{n}{m-1}\binom{N-n}{x-m}(n-m+1)}{\binom{N}{x-1}(N-x+1)},$$ estimation, truncation: *Kungl. Lantbuck. Ann. 18:123.*

4.12. Generalized Poisson

$$D(x)=e^{-\lambda}\sum_{j=1}^{\alpha}\frac{\lambda^{\alpha x+j-1}}{(\alpha x+j-1)!}$$

Connected with Type III: [o]3:123, [o]11:101.

5. Distributions on (*a*, *b*)

5.1. Serial Correlation

$$D(x)=\frac{\Gamma(\frac{1}{2}k+1)}{\Gamma(\frac{1}{2}k+\frac{1}{2})\Gamma(\frac{1}{2})}\,(1-x^2)^{\frac{1}{2}(k-1)}(1+r^2-2rx)^{-\frac{1}{2}k}$$

$$\alpha_1=\frac{rk}{k+2},$$

$v=(k+2)^{-1}[1-r^2k(k+1)(k+2)^{-1}(k+4)^{-1}]$: [c]41:261.

Moments, called "Leipnik": [c]44:270.

See also: [c]35:255, 261, [d]13:1, 14, [c]43:161, 169, [d]18:86, *Cowles Commission Papers, New Series No. 42.*

5.2. Type I

$D(x)=C(x-a)^{p-1}(b-x)^{q-1}$, $a<x<b$, $p>0$, $q>0$: [1]249, [17]No. 22.

Beta for $a=0$, $b=1$ normal for $p=q=\frac{1}{2}b^2$, $a=-b$, $b\to\infty$: [2]139.

Type III for $b\to\infty$, $q=\alpha b$: [1]249.

Obtained by assuming roots in quadratic of Pearson differential equation real with different sign: [4]74, [11]43.

Relations of various constants: [11]53, [c]16:107.

Bayes' theorem: [n]16–1:115.

Fitting to observations: [a]96:306, [c]1:31, [c] 1: 292, [c]1:408, [c]4:474, [c]7:87.

Early volumes of [c] give many examples with $D(x)=C(1+x/a)^m(1-x/b)^n$.

See also: [d]1:148, [d]7:20, [a]8:17, [d]10:15, [c]3:311, [c]23:393, [c]26:386, [i] 16:53.

5.3. Beta (p, q)

$D(x)=\frac{\Gamma[\frac{1}{2}(p+q)]}{\Gamma(\frac{1}{2}p)\Gamma(\frac{1}{2}q)}\ x^{\frac{1}{2}p-1}(1-x)^{\frac{1}{2}q-1}$, $0<x<1$, $p>-2$, $q>-2$: [2]139, [17]No. 21, [1]243, [10]153.

Called Beta distribution of the first kind: [d]25: 401.

$\alpha_1=p/(p+q)$, $v=pq/[(p+q)^2(\frac{1}{2}p+\frac{1}{2}q+1)]$: [2]31, 41, [10]153, [14]42.

3d Moments: [2]419.

Mode $=(p-2)/(p+q-4)$, etc.: [m]2:128.

$\alpha_r=B(p+r, q)/B(p, q)$: [6]117, [b]4:126, [14]42.

$HM=(p-2)/(p+q-2)$, [10]163.

$GM=\exp\{(\partial/\partial\frac{1}{2}p)[\log\Gamma(\frac{1}{2}p)-\log\frac{1}{2}(p+q)]\}$.

C(x): [d]20:451.

Moments: [3]211.

Obtained as a Pearson Type: [4]74.

From an example: [10]45.

C(x): [c]19:1, [c]25:379, [c]38:423, [c]37:208, [i] 38: 192.

$D(\bar{x})$: [2]251, [n]8–4:55.

When $p=q$: [c]19:230.

Special cases and variants: [2]26.

Beta $(n-k, k)$: [1]409.

Beta $(2n-2, 4)$ connected with tolerance limits: *Hoel, Intro. to Math. Stat.*

Correlation ratio in samples from uncorrelated bivariate normal is Beta $(k-1, n-k)$: [2]352, [1]414, [c]21:1, [c]30:290, [10]181, 184.

$D(1-x)=$ Beta (q,p)

If x is Beta (p,q), y is Type III $(1, p+q)$ then xy is Type III (1,p): [10]156.

Standardized Beta variable is N(0,1) as $p\to\infty$, $q\to\infty$: [1]252.

D(xy): [e]9:365.

No sufficient statistic for $(\frac{1}{2}p-1)$: *Proc. Roy. Soc. Lond. (Series A) 154:133.*

Mellin transform: [d]19:373.
more generally: [b]8:136.

$\log\log\frac{1}{x}$, $\arcsin\sqrt{x}$,: [v]8:71.

Generalized: [c]43:237, [c]44:441.

Transform: [c]36:165.

Connection with Fisher and Snedecor: [2]419, [c] 21:350.

Noncentral Beta: [d]26:648.

Connection with binomial: [c]41:304, [n]18:121.

Fitting straight lines: [y]24:23.

Approximated by Poisson: [j]20:19.

Range of rectangular $(0,a)$ is $D(x)=n(n-1)a^{-n}x^{n-2}(a-x)$: [4]93, [c]25:417.

In trivariate normal analysis: [3]341.

GM $(x_1, \ldots, x_n)$ and GM $(1-x_1, \ldots, 1-x_n)$ are joint sufficient: [3]49.

In rank correlation: [2]418.

See also: [d]16:98, [d]17:325, [b]1:214, [c]22:284, [c]22:391, [c]23:143, [c]27:415, [10]154, [c]30:140, [c] 33:178, [c]34:368, [c]35:19, [c]32:151, 271, [c]36:166, [k]5:75, [c]59:204, [c]37:219, [c]40:281, [g]48:831, [w]1:9.

5.4. Type II

$D(x)=[1-(x-m)^2a^{-2}]^p/[aB(\frac{1}{2},p+1)]$, $m-a\leq x\leq m+a$: [2]141.

Properties: *MR10:131.*

Transform to Student: [c]28:308.

$D(\bar{x})$: [n]10–3:91.

C(x): [c]19:12, [c]25:379.

~Dist. of rank correlation coefficient (Pitman): [c]30:259.

Called Thompson's distribution, relation with Student's, normal approximation: [d]27:784.

m location, a scaling, p shape, likelihood function: C.–R.(m) Tables. $v=a^2/(2p+3)$: [d]3:86.

If $m=0$
Distribution of Spearman's ρ for large n: [2]401.

A numerical example: [11]62.

Estimation of center: [t]4:33.

From Pearson system: [11]43.

See also: [c]4:174, [c]16:114, [c]21:263, [n]12–3: 67.

5.5 Partial Correlation

$D(x)=\pi^{-\frac{1}{2}}\frac{\Gamma[\frac{1}{2}(n-k+1)]}{\Gamma[\frac{1}{2}(n-k)]}(1-x^2)^{\frac{1}{2}(n-k-2)}$,

$-1<x<1$: [1]412, [i]24:198.

Type II with $m=0$, $a=1$, $p=\frac{1}{2}(n-k-2)$.

Ch(x): [17]No. 19.

If k=2, transform to Student: [5]99.

If corresponding population parameter is zero: [10]256.

As No. 5.1 with r=0: *Cowles Commission Papers, New Series No. 10.*

If population is non-normal: [i]36:16.

See also: [d]18:81, [n]2:684, [o]3:45, [a]92:580.

5.6. Parabolic

$D(x)=[3(a^2-x^2)]/4a^3$, $-a<x<a$, $v=a^2/5$, grouping corrections: [c]39:432.

Estimation: [d]26:505, [m]6:120.

5.7. Type IX

$D(x)=\frac{m+1}{a}(1+x/a)^m$, $-a\leq x\leq 0$:

[2]142, [17]No. 16

See also: [d]7:26, [c]24:234,240,263.

Type VIII for negative *m*: [17]No. 23, MR4:21.

5.8. Type XII

$D(x)=(p/q)^m\frac{(1+x/p)^m(1-x/q)^{-m}}{(p+q)B(1+m,\,1-m)}$

$|m|<1$, $-p\leq x\leq q$: [2]143.

See also: [d]7:27, [17]No. 24.

5.9. Correlation Determinant

$D(x)=\frac{\Gamma[\frac{1}{2}(n-1)]^{k-1}x^{\frac{1}{2}(n-k-2)}}{\pi^{\frac{1}{2}k(k-1)}\Gamma[\frac{1}{2}(n-2)]\cdot\ \ldots\ \cdot\Gamma[\frac{1}{2}(n-k)]}$

$\alpha_i=(n-1)^{1-k}(n-2)(n-3).\ \ldots\ .\ (n-k)$,

$v=k(k-1)n^{-2}+0(n^{-3})$: [1]411, [4]120.

Downton calls this "Geometric," and mentions, in connection with LSE, the following special cases: [d]25:304.

I. $D(x)=px^{p-1}$, $0\leq x<1$,

$C(x)=x^p$, $\alpha_i=p/(p-1)$, $v=p(p+2)^{-1}(p+1)^{-2}$

II. $D(x)=pb^{-p}(x+a)^{p-1}$, $-a\leq x<b-a$

III. $D(x)=pv^{-\frac{1}{2}}b^{-p}[(x-m)/v^{\frac{1}{2}}+a]^{p-1}$,

$m-av^{\frac{1}{2}}\leq x<m+(b-a)v^{\frac{1}{2}}$, $\alpha_1=m$,

$a=p^{\frac{1}{2}}(p+2)^{\frac{1}{2}}$, $b=p^{-\frac{1}{2}}(p+1)^{3/2}$, $p\geq 1$.

log log (x): [v]8:70.

5.10. Triangular

$D(x)=1-|1-x|$, $D(\bar{x})$ from rectangular when n=2, D(range): [c]25:417, [y]24:22, MR 3:171.

$D(x)=\frac{2x}{2k+1}$, $k\leq x\leq k+1$: [3]47, [8]32.

Stratified sampling: [c]13:48.

$D(x)=(9\sigma)^{-1}\left[\frac{x-m}{\sigma}+2\sqrt{2}\right]$, right triangular

$m-2\sqrt{2}\sigma\leq x<m+\sqrt{2}\sigma$: [d]4:256, [d]25:308.

$D(x)=4R^{-2}(\frac{1}{2}R-|x-m|)$, $|x-m|\leq\frac{1}{2}R$, best linear estimate of m and σ: [d]25:318.

Testing: [d]25:695.

See also: [d]2:48, [d]28:179.

$D(x)=\frac{1}{a}\left[1+k-\frac{2k}{a(a-x)}\right]$, $0\leq x\leq a$,

$-1\leq k\leq 1$. called "linear": [d]4:244.

$D(x)=\begin{cases}1+x, & -1\leq x<0\\ 1-x, & 0\leq x<1\end{cases}$, called "Tine": [d]5:33.

$Ch(x)=(2/t^2)(1-\cos t)$: [17] No. 14.

$D(x)=\frac{2}{a^2}(a-x)$, called "semi-triangular"

$m=a/3$, $v=a^2/18$, grouping corrections: [c]39:432.

Ch(x): [17] No. 13.

Triangular on (a, b). If x and y are extreme values of the sample, then

$E[\frac{1}{2}(x+y)]=\frac{1}{2}(a+b)$,

$\text{Var}\ [\frac{1}{2}(x+y)]=\frac{4-\pi}{16n}(b-a)^2+0(n^{-2})$,

$E[(x-y)]=\left[1-\frac{\sqrt{\pi}}{2n}\right](b-a)+0(n^{-3/2})$,

$\text{Var}\ (x-y)=\frac{4-\pi}{4n}(b-a)^2+0(n^{-2})$.

See also: [y]16:16.

5.11. Rectangular $(a-h,\ a+h)$

$D(x)=\frac{1}{2h}$, $a-h\leq x\leq a+h$; $D(x)=0$ elsewhere

$m=a$, $v=h^2/3$: [1]244, [5]34.

Skewness=0, kurtosis=$-6/5$, K_s, Ch(x): [18]1-136, 1-144.

$Ch(x)=(\sin ht/ht)e^{ait}$: [1]259.

Special case of Type II: [2]142.

Ch(x), bibliography for rectangular over (a,b), (−a, a): [17] *Nos. 11, 12.*

$C(x)=\frac{1}{2}+x/a$: [15]*93.*

If x and y are the k^{th} values from the top and bottom of sample,

$$E(x)=a+h-\frac{k}{n+1}(2h);\quad E[\tfrac{1}{2}(x+y)]=a;$$

$$E(x-y)=\left(1-\frac{2k}{n+1}\right)(2h):\quad [1]372.$$

$$\text{Var }(x)=\frac{k(n-k+1)}{(n+1)^2(n+2)}(2h)^2$$

$$\text{Var }[\tfrac{1}{2}(x+y)]=\frac{4kh^2}{2(n+1)(n+2)}$$

$$\text{Var }(x-y)=\frac{2k(n-2k+1)}{(n+1)^2(n+2)}(2h)^2$$

$$\sim D(\bar{x})=N\left(0,\frac{h^2}{3n}\right),\ \sim D(c)=\text{Laplace}\left(0,\frac{h}{n}\right),$$

where $c=\frac{1}{2}$ (max+min), n var (c)=6 var (x): [3]*48.*

$D(\xi)$: [d]*26:115.*

Moments of max and min: *MR 4:21.*

$D(\bar{x})$ given incorrectly: [n]*10–3:91.*

$D(\bar{x})$=No. 5.16.

$D(-2\log \Pi x_i)$: [c]*4:161.*

FD(a)=Rectangular(max−h, min+h): [c]*30:402.*

$D(q^{th}$ ranking item)=Type I: [c]*23:390.*

Testing against simple unimodal distribution: [y]*20:111.*

Grouping corrections: [c]*39:430.*

Transformation to Cauchy: [15]*101.*

C.−R. theorem may not hold: [1]*485.*

MLE(a−h, a+h)=(max, min): [6]*156*, [3]*28.*

Best linear estimate of m and σ: [d]*25:308,317.*

Estimation: [d]*17:355.*

Location and scaling, closest estimate: [u]*33:221.*

Minimax estimate of "a": [d]*22:37.*

UMVUE: [14]*142.*

Bayes' theorem: [n]*16–1:110.*

Variance of estimates of "a": [g]*36:410.*

Testing "a": [d]*25:157.*

Critical regions: [3]*280.*

See also: *Archiv. der Math. 3:3, MR 6:235.*

5.12. Rectangular (0, a)

D(x)=1/a, Ch(x)=$(e^{ait}-1)/ait$: [2]*245.*

MGF(x)=$(\sinh \frac{1}{2}at)/\frac{1}{2}at$: [10]*38.*

$\alpha_{2r}=(\frac{1}{2}a)^{2r}/(2r+1)$: [10]*14.*

Cumulants: [10]*41.*

Var(mean deviation)$\simeq a^2/45n$: [2]*217.*

Mean difference: [c]*28:432.*

Ch(Σx_i), C(Σx_i): [9]*278.*

$$D(GM)=\frac{n^n x^{n-1}}{a^n\Gamma(n)}(\log a/x)^{n-1}:\quad [2]246,[d]5:276.$$

$$D(\text{range})=n(n-1)a^{-n}x^{n-2}(a-x):$$
[4]*92, 123*, [c]*20A:210*, [9]*241.*

$$D\left(\sum_{j=1}^{n}\log x_j\right)=\text{Type III}:\quad [c]7:296.$$

$D(\bar{x}, s)$ for n=2,3: [d]*3:128.*

D(quotient of ranges): [g]*46:502.*

$D(\max_1/\max_2)$=No. 8.63: [g]*50:1136.*

$FD(a)=k\,x^{-n-1}$: [c]*30:408*, [d]*9:273.*

MGF $\left(\log\log\frac{1}{x}\right)$, MGF (arcsin $\sqrt{x}$): [c]*8.69.*

Completeness: [c]*10:314.*

Estimation by order statistics: [d]*26:576.*

Quasi-range: [d]*28:179.*

Best linear estimate: [d]*14:88.*

$$\text{UMVUE }(a)=\left(1+\frac{1}{n}\right)(\max),\ \max \text{ is sufficient:}\quad [14]142.$$

Sufficient statistics: [c]*17:214.*

Confidence intervals for a: [3]*83*, [b]*17:88.*

Example: [d]*11:209.*

Estimation of dispersion: [c]*36:95.*

UMP test of a=1: [13].

See also: [d]*2:48*, [d]*2:66*, [d]*4:126*, [d]*4:139, 142,255*, [c]*23:424.*

5.13. Rectangular (0, 1)

$D(x)=1,\quad 0\leqq x\leqq 1$

$D(-\log x)=e^{-t}$, $Ch(-\log x)=(1-it)^{-1}$

$D(-\Sigma\log x_i)$=Type III (1, k): [3]132.

$$D(\Sigma x_i)=\frac{1}{(n-1)!}[x^{n-1}-\binom{n}{1}(x-1)^{n-1}+\binom{n}{2}(x-2)^{n-1}-\ldots]$$

"Irwin-Hall" distribution: [d]*13:43*, [1]*245*, [w] *1:73*, [2]*240, 244*, [c]*19:234*, *MR 12:509*, *MR 15:42*, *MR 7:311*, [c]*19:240*, [c]*41:334.*

Obtained by finite differences (Also for No. 4.4): [p]*10:52, 255, 324.*

Convolutions: *MR 6:88.*

Sheppard's corrections: [2]*88.*

D(q^{th} value from top of sample)=Type I, $m=1-q/(n+1)$, $v=q(n-q+1)/[(n+1)^2(n+2)]$: *[2]218.*

Var $(\xi)=1/4(n+2)$: *[2]230.*

$\sim D(\bar{x})$: *[d]25:636, MR 9:360.*

Mellin transform: *[d]19:373.*

Order statistics: *[c]24:260, [i]33:214.*

D(range)=Type I: *[c]25:417.*

MGF(log log $1/x$)$=(1+t)$
MGF(arcsin x)$=2(e^{\frac{1}{2}\pi t}+1)/(t^2+4)$: *[v]8:69.*

Ratio of two ranges: *[d]21:112, [x]7:179.*

Moments of the range: *Z 13:30, [c]20:217.*

C.–R. theorem may not hold: *[1]485.*

Censored sample: *[c]41:230.*

Stratified sample: *[d]13:44.*

Significance levels for $\bar{x}$: *[t]3:172.*

D(GM)=No. 8.12: *[w]1:73.*

Estimation of center, $D(\bar{x})$, $D(\xi)$: *[c]33:126.*

$D(\max_1 \max_2)$=No. 8.64: *[g]*50:1142.

Two rectangulars added: *[n]8-3:74.*

Hypothesis testing: *[c]32:321.*

See Also: *[d]23:43, [c]25:203, [m]6:120, [d]22:418, [y]24:21, MR 7:310, Z 11:218, MR 16:602.*

5.14. Correlation

$$D(x)=\frac{(1-x^2)^{\frac{1}{2}(n-4)}(1-\rho^2)^{\frac{1}{2}(n-1)}2^{n-3}}{\pi(n-3)!}\sum_{i=0}^{\infty}\frac{(2x\rho)^i}{i!}\Gamma^2\left(\frac{n+i-1}{2}\right)$$ *[1]398.*

$$D(x)=\frac{(1-\rho^2)^{\frac{1}{2}(n-1)}(1-x^2)^{\frac{1}{2}(n-4)}}{\pi(n-2)!}\frac{d^{n-2}}{d(\rho x)^{n-2}}\left[\frac{\cos^{-1}(-\rho x)}{1-\rho^2x^2}\right]$$ *[2]342, [10]200.*

Special cases n=2,3,4, moments: *[2]345.*
n=4: *[u]26:536.*

C(x): *[c]25:71.*

$\sim D(x)$: *[n]1-4:1, [c]38:236.*

If $\rho=0$, D(x)=No. 5.5: *[6]314, [4]120, [g]26:129.*

If $\rho=0$, $D\left(\frac{x}{\sqrt{1-x^2}}\sqrt{n-2}\right)$=Student(n−2): *[2]343.*

Transform $x=\tanh z$, $\rho=\tanh \zeta$: *[10]200, [c]21:358.*

If $\rho=0$, $D(x^2)$=Beta $[\frac{1}{4}, \frac{1}{4}(n-2)]$: *[10]160, 192.*

Bayes' distribution of ρ is No. 5.5(0,1): *[3]91, [c]41:278.*

Moments: *[n]5:3.*

Papers dealing with this distribution generally: *[c]10:507, [c]11:328.*

Interval estimation: *[e]7:415.*

Confidence limits for ρ: *[c]29:157.*

Stratified sampling: *[i]36(Suppl.):87.*

See also: *[b]15:193, [c]21:164, [c]24:383; [o]3:1, Z21:41.*

5.15. Multiple Correlation

$$D(x)=\frac{\gamma\,(1-R^2)^{\frac{1}{2}(n-1)}}{B[\frac{1}{2}(n-k),\,(k-1)]}x^{\frac{1}{2}(k-3)}(1-x)^{\frac{1}{2}(n-k-2)}$$

where $\gamma=F\left(\frac{n-1}{2}, \frac{n-1}{2}, \frac{k-1}{2}, R^2x\right)$ and

$$F(a, b, c, x)=1+\frac{ab}{c}x+\frac{a(a+1)b(b+1)}{c(c+1)}\frac{x^2}{2!}+\cdots:$$

[14]65, [2]384, [d]3:196, [2]127.

If R=0, $D(x^2)$=Beta(k, n−k): *[i]30:63, [2]381*
Testing: *[i]29:25*

Another form: *[2]387, [3]338.*

Limiting form when $n\to\infty$: *[2]387.*

When R=0, D(x)=Snedecor: *[10]257,252.*

Mean, variance: *[c]22:353.*

Moments: *MR 14:189.*

More generally: *[d]11:6.*

See also: *[e]9:352, [e]10:257, [a]92:445, [n]12-4:67, [i]24:199, [x]1:67,137, [x]4:88, Proc. Roy. Soc. (A) 121:654, [u]46:521.*

5.16. Rectangular Mean

$$D(x)=\frac{1}{(n-1)!}\sum_{j=0}^{[x]}(-1)^j\binom{n}{j}(x-j)^{n-1}:$$ *[1]245.*

Generalization: *[e]3:330.*

Obtained by finite differences: *[p]10:52, 255, 324.*

Called Irwin-Hall, cf. No. 5:13 *[d]13:43.*
Compare No. 8.47 and No. 8.70:

See also: *London P.O. Res. Rep. 13443, Archiv der Math. 3:3, Proc. Intl. Cong. Math. (1924) 2:795.*

6. Distributions on (0, ∞)

6.1. Type VI

$D(x)=C(x-a)^{p-1}(x-b)^{q-1}$, $x>b$, $a<b$, $q>0$, $p+q<1$: *[1]249.*

If b=0, $C=\frac{a^{1-p-q}}{B(1-p-q,\,p)}$: *[2]140.*

Roots of quadratic in Pearson equation real and same sign: *[11]45.*

Truncation: [i]39:63, [i]40:18.

Various constants and an example: [11]83.

See also: [d]7:23, [c]23:143, [c]25:379, [17]No. 26, [c]4:167.

6.2. Snedecor (p, q)

$D(x)=\frac{(p/q)^{\frac{1}{2}p}x^{\frac{1}{2}p-1}(1+px/q)^{-\frac{1}{2}(p+q)}}{B(\frac{1}{2}p,\frac{1}{2}q)}$, $x>0$: [5]100.

"F" distribution

m, v, $\kappa_3,\kappa_4,\beta_1,\beta_2$: [p]6:175.

Derivation, properties, examples: [15]374, [18]1–163.

Area unity if p,q both even: MR 12:509.

Obtained as distribution of ratio of two Chi-square variables: [6]10.5, [4]113.

$1-C(x)=\frac{\sum_0^n c_j x^j}{(x+k)^m}$, where $n=\frac{1}{2}p-1$,

$m=\frac{1}{2}(p+q)-1$, $k=q/p$, and $c_j=\binom{m-1}{j}k^{m-j-1}$: [p]10:62.

$D(\sqrt{x})$: [5]100.

$D\left(\frac{px}{q+px}\right)=B(p-1,\ q-1)$: [4]115

Therefore called "inverted Beta": [c]33:73.

Various properties: [d]12:446.

$m=\frac{q}{q-2}$: [10]198.

$\alpha_r=\frac{\Gamma(\frac{1}{2}p+r)\Gamma(\frac{1}{2}q-r)}{\Gamma(\frac{1}{2}p)\Gamma(\frac{1}{2}q)}(q/p)^r$: [4]114.

$\text{Mode}=\frac{pq-2q}{pq+2p}$: [10]197.

Approximated by normal distribution: [d]13:233.

If x, y each Snedecor (n-1, n), then $D(\sqrt{x/y})$= Snedecor (2n–2, 2n–2).

Testing: [d]13:371.

Used to test multiple correlation coefficient: [10]257.

See also: [4]189, [d]6:204, [d]18:89, [c]21:350, [c]37:219, [q]7:96, J. Soc. Statist. Paris 96:262.

6.3. Beta of Second Kind (p, q)

$D(x)=\frac{x^{p-1}}{B(p,q)(1+x)^{p+q}}$, $m=\frac{p}{q-1}$,

$v=\frac{p(p+q-1)}{(q-1)^2(q-2)}$, $\text{mode}=\frac{p-1}{q+1}$, $HM=\frac{p-1}{q}$

if $r<q$, $\alpha_r=\frac{p(p+1)\ldots(p+r-1)}{(q-1)(q-2)\ldots(q-r)}$

$D(1/x)$=No. 6.3(q,p), $D\left(\frac{1}{1+x}\right)$=Beta: [1]242, [10]156, 158, 163 [d]25:402.

$p=q$, $x\geqq 1$: [d]22:418.

Called Fisher's F: [w]1:9.

C(x): [p]7:102.

6.4. Hotelling

$D(x)=\frac{2}{B[\frac{1}{2}(p-q),\frac{1}{2}q](p-1)^{\frac{1}{2}q}}\frac{x^{q-1}}{\left[1+\frac{x^2}{p-1}\right]^{\frac{1}{2}p}}$: [4]238.

For $q=1$, this is positive half of Student distribution, hence called generalized Student: [1]409, [d]2:375.

$D(x^2)$: [c]32:70.

Mellin transform: [d]19:373.

Percentage points, relation with Chi-square: [d]27:1091.

See also: [10]207, [c]25:399, [i]30:66, [d]9:235, [c]24:480, [c]4:174, [c]24:487, [t]7:82.

6.5. Pareto

$D(x)=p/q(q/x)^{p+1}$: [8]120.

$\alpha_1=\frac{p}{p-1}q$, $\xi=2^{1/p}q$: [1]248, [2]142.

More generally: [d]7:26.

Testing, location and dispersion: [t]7:115.

As Type XI: [c]39:178, [17]No. 25.

Ranking: [c]24:234, 241, 275.

Double Pareto: *Kendall and Buckland, A Dictionary of Stat. Terms.*

See also: [l]19:174, [h']1:149, [g]48:537, [i]8:76, [t]3:77, [y]13:30, MR13:962, Z23:63, C. R. Acad. Sci. Paris 233:1421, [l]25:591, [w]4:147.

6.6. Kendall

$D(x)=\frac{re^{-(x-r)/\alpha}(x-r)^{x-1}}{\alpha^x\Gamma(x+1)}$, $0<r<x<\infty$, $0\leq\alpha\leq 1$,

$m=\frac{r}{1-\alpha}$, $v=\frac{r\alpha^2}{(1-\alpha)^3}$.

[b]19:211, (*cf. Borel–Tanner*).

6.7. Inverse Gaussian

$D(x)=\exp\ [-\lambda(x-\mu)^2/2\mu^2x][\lambda/2\pi x^3]^{\frac{1}{2}}$, introduction, moments, estimation: [d]28:362, 696.

7. Distributions on $(-\infty, \infty)$

7.1. Type VII

$D(x)=\frac{(1+x^2/a^2)^{-m}}{aB(\frac{1}{2}, m-\frac{1}{2})}$, $m>\frac{1}{2}$: [2]142.

Estimation: [c]36:412, [t]4:35.

See also: [c]15:401, [c]36:412, 167.

7.2. Student (ν)

$D(x)=\frac{\Gamma[\frac{1}{2}(\nu+1)]}{\Gamma(\frac{1}{2}\nu)(\pi\nu)^{\frac{1}{2}}}(1+x^2/\nu)^{-\frac{1}{2}(\nu+1)}$: [5]97, [2]17, [6] 10.6, MR8:161, [14]47, [4]110, [10]186, [l]3:355, [18]1–162, [17]No. 29.

Introduction, properties, examples: [15]388.

$V=\nu/(\nu-2)$. [1]239.

Type VII with $m=\frac{1}{2}(\nu+1)$, $a^2=\nu$. "t" distribution.

$\alpha_{2k}=\frac{1\cdot 3\cdot \ldots \cdot (2k-1)\nu^k}{(\nu-2)(\nu-4)\cdot \ldots \cdot (\nu-2k)}$: [1]239, [10]208.

Original paper in which this distribution was discovered: [c]6:1.

Ch(x), refs for $\nu=3$: [17]No. 28.

Ch(x): [b]18:212.

Distribution of the ratio of a Chi-square variable to a normal variable: [1]387, [4]110, [10]187 [n]5:102.

In bivariate normal samples

$D\left(\frac{\sigma_1(n-1)^{\frac{1}{2}}}{\sigma_2(1-\rho^2)^{\frac{1}{2}}}(b-\beta)\right)=$ Student $(n-1)$: [1]29.8.

$D\left(\frac{s_1(n-2)^{\frac{1}{2}}}{s_2(1-r^2)^{\frac{1}{2}}}(b-\beta)\right)=$ Student $(n-2)$: [1]29.8.

if $\rho=0$

$D\left[(n-2)^{\frac{1}{2}}\frac{r}{(1-r^2)^{\frac{1}{2}}}\right]=$ Student $(n-2)$: [1]29.7.

C(x): [l]3:358, [c]25:389, [n]5:109, [c]37:168.

$D(x^2)=$ Snedecor: [6]217, [4]115.

Transform to Type II: [c]28:308.

$D(\bar{x})$: [n]8–4:92.

As $\nu\to\infty$, Student$\to N(0,1)$: [1]252, [3]101, [a] 113:228, [d]27:783, Proc. A.M.S. 6th Symposium in Appl. Math. p. 251.

Approximations: [d]7:210, [d]9:87, [d]17:216.

D(log x): [c]34:176.

Two Student variables: [c]22:405, [c]23:1.

Used to test partial correlation: [10]256.

~significance levels: [d]14:60.

Generalizations: [d]19:406, [d]25:162, [i]34:58.

See also: [d]10:265, [d]18:89, [e]11:37, [e]12:89, [j]8:632, [c]33:362, [n]5:90, [c]32:271,300, [c]24:56,296, [i]33:138, [c]44:264, Brit. Assoc. Math. Tables (3rd Ed.) V. 1 p xxxiii, J. Soc. Stat. Paris 92:262, MR18:834, Z4:67, [u]21:482,655, [w]1:9, [p]8:42.

7.3. Normal Regression Slope

$D(x)=\frac{[\sigma_1^2\sigma_2^2(1-\rho^2)]^{\frac{1}{2}(n-1)}\Gamma(\frac{1}{2}n)}{\sqrt{\pi}\Gamma[\frac{1}{2}(n-1)]\sigma_1^{n-2}(\sigma_2^2-2\rho\sigma_1\sigma_2 x+\sigma_1^2x^2)^{\frac{1}{2}n}}$: [1]402.

$v=\frac{1}{n-3}\frac{\sigma_2^2}{\sigma_1^2}(1-\rho^2)$: [2]365, [e]1:432.

For $\rho=0$, can use Student distribution to test x: [10]194.

Stratified sampling: [i]36:96.

7.4. Cauchy (p, q)

$D(x)=\frac{1}{\pi}\frac{p}{p^2+(x-q)^2}$: [1]246, [5]35, [18]1–36.

$Ch(x)=\exp[qit-p|t|]$: [5]60.

q is the mode and median; there is no mean, nor any moment: [5]58.

Quartiles are $q\pm p$: [5]67.

$\bar{x}$ is not a consistent estimate of q: [5]105.

$\bar{x}$ is a "density unbiased" estimate of q: [d]25:400.

There are no sufficient estimators: [3]48.

C.–R. (q), C.–R. (p), C.–R. (p/q): [e]8:205.

Distribution of t and F statistics: MR13:665.

Mean and variance of $\frac{1}{2}(x+y)$, where x and y are respectively the k^{th} values from the top and bottom of the sample: [1]373.
$\frac{1}{2}(x+y)$ is not a consistent estimate of q.

See also: [d]17:2, [d]21:133.

7.5. Cauchy (1, q)

$D(x)=\frac{1}{\pi}\frac{1}{1+(x-q)^2}$: [6]117.

q incorrectly asserted to be the mean: [p]7:165.

E(x), $E(x^2)$, D(x+y): [14]43.

C.–R. (q)$=2/n$: [1]490, [3]24, [p]7:159.

Var $(\xi)\cong\pi^2/4n$: [3]6.

$D(\bar{x})=D(x)$, hence $\bar{x}$ not consistent: [3]2, [1]490, [u]22:702.

$D(\xi)$: [3]46.

MLE $\neq$ minimax: [16]64.

MLE is solution of $\Sigma[2(x_1-q)]/[1+(x_1-q)^2]=0$: [3]24, [p]7:169.

Gaging: [e]15:194.

There is no sufficient estimator: [9]6:16, [3]27, [p]7:162.

There is no UMVUE: [3]51.

Information and estimation: [e]8:315.

Loss of information: [3]32.

Testing $m=m_0$: [d]9:83, [d]13:65.

Cauchys added: MR17:863.

See also: [b]9:61, [i]20:61, [g]51:641, [d]28:832, [w]1:9.

7.6. Cauchy (p, 0)

$D(x)=(p/\pi)[1/(p^2+x^2)]$, $Ch(x)=\exp -p|t|$: [9]275.

Reproductive property: [9]276.

Information and estimation: [e]8:316.

Completeness: [e]10:314.

$D(\bar{x})=D(x)$: [n]10–3:91.

Truncated to $(-p,p)$: [10]14.

7.7. Cauchy (1, 0)

$D(x)=(1/\pi)[1/(1+x^2)]$ [8]167.

$Ch(x)=\exp -|t|$: [2]95, [8]167, [1]246, [9]243, [17] No. 27, [s]213:718, [i]5:133.

Sample median: [d]26:600.

$D(\bar{x})=D(x)$: [2]233,247, [w]1:73.

From an example: [8]33, [9]242.

Moments: [8]99.

As distribution of ratio of two normal variables: [10]159.

$C(x)=\frac{1}{2}+(1/\pi)\tan^{-1}x$.

Censored sample: [c]41:230.

Wrapped-up Cauchy: [d]26:245.

See also: [d]22:425, [k]7:371, *S. D. Poisson (1824) "Sur la Probabilité des résultats moyens des observations," Connaissance des Tems ou Des Mouvements Célestes a l'usage des astronomes et des Navigateurs, pour l'an 1827. Le Bureau des Longitudes, Paris.* [d]22:418, MR3:232, MR9:235, MR5:124.

7.8. LaPlace (m, v)

$D(x)=(2v)^{-\frac{1}{2}}\exp[-|x-m|/\frac{1}{2}(2v)^{\frac{1}{2}}]$: [1]247, [5]35, 67, [18]1–136.

$Ch(x)=\exp(mit)\cdot(1+\frac{1}{2}vt^2)^{-1}$: [5]62.

Mean and variance of average of greatest and least sample values: [1]375.

A priori distributions of m,v: MR9:294.

"Best" estimates of m and σ are ξ and $(1/n)\Sigma|x_i-\xi|$, but ξ not sufficient: [5]147–8.

Distribution of smallest sample value: [g]43:408.

$D(\xi)$: [d]26:115.

Quasi-range: [d]28:179.

"Double exponential" distribution: *Proc. Roy. Soc. Lond. (Series A) 154:124,* [v]7:164.

Convolution, estimation, generalization: *Sobre la Primera Ley de Errores de Laplace, F. A. Sales Vallés, Thesis, Barcelona 1947.*

Laplace (0, v), also called "Poisson's first law of error," $D(|\bar{x}|)$, $D(|x|-|y|)$, $D(\log|x|-\log|y|)$, $D(\Sigma x_i^2)$, D(GM), D(HM): [d]6:102.

Laplace if $D(x)D(y)=\phi(|x|+|y|)$ MR10:125.

See also: [n]10–3:80.

Laplace (m,2), MLE: [3]45.

Laplace (0,1), $D(x)=e^{-|x|}$: [8]120, [9]279, [1]100.

Sample median: [d]26:599.

$\alpha_{2k}=(2k)!$: [d]5:32.

D(range), quality control: [p]8:87.

See also: [d]22:425, [17] No. 38, *Laplace (1774), "Memoire sur la Probabilité des causes par les évènemens."*

7.9. Fisher (p, q)

$$D(x)=\frac{2p^{\frac{1}{2}p}q^{\frac{1}{2}q}e^{px}}{B(\frac{1}{2}p,\frac{1}{2}q)(q+pe^{2x})^{\frac{1}{2}(p+q)}}$$: [1]243, [2]249, [14]48, MR8:161.

z distribution: [l]3:355.

C(x) as a power series: [l]3:360.

Cumulants: MR9:48, 735.

Moments, cumulants: [y]5:317.

Transform to Snedecor by $y=e^{2x}$; $x=\frac{1}{2}\log qx_1^2/px_2^2$: [c]23:147.

Various properties: [d]12:429, [c]34:173.

$$Ch(x)=\left(\frac{q}{p}\right)^{\frac{1}{2}it}\Gamma[\tfrac{1}{2}(q-it)]\Gamma[\tfrac{1}{2}(p+it)]/\Gamma(\tfrac{1}{2}p)\Gamma(\tfrac{1}{2}q)$$: [3]116.

$\sim C(x)$: [d]28:504.

Obtained from two Chi-square variables: [2]249, [d]7:52, [g]26:173.

Approximate significance levels, transformation to normal: [d]11:93.

Normal limit: *Am. Math. Monthly, 50:100.*

Generalization: [i]34:58.

Noncentral: [o]7:57.

See also: [e]2:423, [b]1:31, [a]94:284, [c]21:350, (c)34:352, 359, [c]41:304, *Am. Math. Monthly 50:100,* 382. *Proc. Intl. Cong. Math. (1920)805, Current Science 1941, p. 191,* [w]5:30, [p]6:183.

7.10. Type IV

$D(x)=C(1+x^2/a^2)^{-m}\exp(-p\tan^{-1}x/a)$: [11]69.

$D(x-h)=$Type IV: [3]48.

Various constants with an example: *[11]69.*

Roots of quadratic in Pearson equation complex: *[11]44.*

$\alpha_r=[a/(2m-r-1)][(r-1)a\alpha_{r-2}-p\alpha_{r-1}]$: *[2]86, 140, 144.*

$a=1$: *[17]No. 52.*

See also: *[d]7:21, [c]1:39, [c]3:312, [c]6:435, [c]7:74, [c]26:386.*

8. Miscellaneous Univariate

8.1. Pearson

$$\frac{1}{D(x)}\frac{dD(x)}{dx}=\frac{a_0+a_1x}{b_0+b_1x+b_2x^2},$$

types listed with associated parameters: *[d]2:394, [n]11–4:77.*

Differential equation for Ch(x): *MR8:393.*

Ch(x): *MR10:705.*

Original paper: *Phil. Trans., 1895.*

First seven types treated in [11].

New classification: *[d]7:16.*

D(x̄): *[d]18:111, [n]8–4:51.*

Truncation, estimation: *[d]22:256, [c]40:50.*

Bivariate: *MR9:363,452.*

Flexes equidistant from mode, etc.: *[d]6:1.*

Bivariate generalization: *[e]3:273.*

Orthogonal Polynomials: *Ann. Soc. Cien. Argentina 155:3.*

Generalizations: *[d]5:124, [c]26:129, (cf No. 8.52), MR10:386, MR17:1095, [d]21:289, J. Gakugei Tokushima Univ. Math. 5:29, [n]12–2:95.*

Log Pearson distributions: *Intl. Cong. Math. (1950)1:580.*

Romanovsky's generalization: *[c]17:106, [c]18:221.*

See also: *[d]8:18, [d]8:206, [d]20:461, [e]6:415, [c]7:127, [c]16:106, [c]16:198, [c]18:264, [c]20:389, [g]26(P):288, [a]85:488, [c]35:113, [c]32:81, [c]36:151, [c]38:4, [i]25:141, MR17:169,272, Z9:314, Z6:268, Z19:73, MR14:755,977, Intl. Cong. Math. (1950) 1:585.*

8.2. Bessel Function

Mahalanobis' Distribution ("D" distribution): *[e]2:143,385, [e]3:105, [c]4:19,373,535, [e]8:167, [k]8:379, [14]246, MR4:23.*

Wilk's distribution of dispersion determinant, etc.: *[e]3:26.*

D(x̄): *MR5:42.*

Distribution of vector correlation: *[c]28:353.*

Distribution of difference of χ^2 variables: *[d]7:51.*

D(x), Ch(x), Moments in a special case: *MR14:775.*

D(ratio Type III variables): *Studies Presented to Richard von Mises. Academic Press, 1954. p. 301.*

Bivariate Gamma distribution: *[e]5:140.*

Distribution of the range: *[d]18:384, [d]21:133.*

Marginal total of Elfving's distribution: *[c]36:142.*

See also: *[e]6:175, [e]8:235, [a]97:125, [a]98:89, [b]16:96, [c]21:168, [c]24:441, [c]24:485,492, [d]18:392, [c]24:39, [c]24:293, [17]Nos. 10,60,61,62,63,64, with Ch(x), refs. [c]21:164, [c]24:293, [c]24:39, Several forms, J. Soc. Stat. Paris 96:262, [u]28:458, MR11:607, MR16:152.*

8.3. Variance Ratio

$$D(x)=\frac{2(1-\rho^2)^{\frac{1}{2}(n-1)}x^{n-2}(1-4\rho^2x^2(1+x^2)^{-2})^{-\frac{1}{2}n}}{B[\frac{1}{2}(n-1),\frac{1}{2}(n-1)](1+x^2)^{n-1}},$$

"Bose", *[2]365, [e]2:65.*

For $\rho=0$, $D[(2n-3)x^2]$=Hotelling $(2n-2,n-1)$.

See also: *[p]6:183, [p]7:98, [c]30:190, [c]31:9.*

8.4. Kullbach

$$D(x)=\frac{nx^{np-1}}{\Gamma(n)[\Gamma(p)]^2}\sum_{j=0}^{\infty}(-1)^{n+nj+1}\left(\frac{d^{n-1}}{dt^{n-1}}\frac{x^{nt}}{\Gamma(t+1)}\right)_{t=j}$$

Distribution of GM from Type III (1,p): *[2]251.*

8.5. Noncentral Student

D(x): *[c]31:362, [18]1–162, [e]4:173,307.*

Multivariate: *[e]4:331.*

Application: *[c]43:219.*

See also: *[i]36(Suppl.):21, [r]1:28, J. Soc. Stat. Paris 96:262, MR15:46.*

8.6. Continuous Lexian

$D(x)=\int_0^1 f(p)\binom{n}{x}p^x(1-p)^{n-x}dp$; parameters; if f(p) is Beta, D(x) is hypergeometric: *[i]31:1, [i]34:197.*

8.7. Noncentral Snedecor

D(x): *[c]36:220, [18]1–163, [e]15:321.*

See also: *[c]38:112, [i]36(Suppl.):33, [r]3:33.*

8.8. Fisher's Logarithmic Series

$$D(x)=\frac{k^x}{p\log\frac{1}{1-k}},\ x=1,2,\ldots:$$ *[c]35:6, [f]5:162.*

Cf. negative binomial (1,−m), No. 3.5: *[c]37:358.*

8.9. Rank Variate

$D(x)=\frac{N}{\sigma}\frac{n!}{(q-1)!(n-q)!}\left[\exp-\frac{x}{\sigma}(n-q+1)\right]\times(1-e^{-x/\sigma})^{q-1}$: [c]24:231, 239, [c]25:79.

In special case called Yule's distribution, MLE: [c]42:23,425, [c]43:248.

A distribution of Type III median: [p]7:153.

8.10. Generalized Pareto

$D(x)=ax^{-n}[1/(e^{b/x}-1)]$: [l]6:184.

$b=1$, $n=5$, $a=15/\pi^4$, Planck's radiation function: [17]No. 56.

8.11. Ghosh

$D(x)=\frac{2}{\Gamma([k]+1)}x^{[k]}e^{-x^2}$, where [k] is the largest integer $\leq k$.

Furnishes counterexample to theorem on similar regions: [e]8:330.

8.12. Rectangular Geometric Mean

$D(x)=\frac{n^n x^{n-1}}{a^n\Gamma(n)}(\log a/x)^{n-1}$: [2]246, [d]5:276, [w]1:73.

8.13. Cauchy Median

$D(x)=\frac{(2m+1)!}{(m!)^2\pi^{2m+1}}\left(\frac{\pi^2}{4}-[\tan^{-1}(x-k)]^2\right)^m\frac{1}{1+(x-k)^2}$; [3]46.

8.14. Spearman's Rank Correlation

$D(x)$: [c]30:256, [c]34:183, [c]38:131.

$\sim D(x)$ = Type II, moments: [c]40:409.

8.15. Circular Normal Correlation

$D(x)=n(n-1)\,e^{-x^2/v}(1-e^{-x^2/2v})^{n-2}(x/\sigma)$: [c]39:139, [g]48:496.

8.16. Koopman

$D(x)=Q(k)\,R(x)\exp k\,H(x)$, most general distribution admitting a sufficient estimate of k: [3]24, [d]23:403, [c]36:71, *Trans Am. Math. Soc. 39:399*, [p]7:162.

8.17. Von Mises

$D(x)=C\exp k\cos(x-a)$: *Physikal. Zeitschr 19:490.*

'Circular Normal': [g]48:131, [g]49:53,268, [c]43:344, [d]26:233, [c]43:344.

8.18.

Family of distributions having all moments equal: [d]11:402.

$D(x)=\frac{1}{6}e^{-x^{\frac{1}{4}}}(1-p\sin x^{\frac{1}{4}})\quad 0\leq x<\infty, 0\leq p\leq 1.$

$\alpha_k=\frac{1}{6}(4k+3)!$.

8.19.

Family of distributions having all moments equal $D(x)=e^{-\frac{1}{4}}\pi^{-\frac{1}{2}}x^{-\log x}[1-p\sin(2\pi\log x)]$, $\alpha_k=\exp[\frac{1}{4}k(k+2)]$: [d]11:402.

8.20.

"Non-null t^2 distribution", involving a hypergeometric function: [14]48.

8.21.

$D(x)=\frac{\operatorname{sech}^{k-2}x}{B(1,k-2)}$, the distribution of $\tanh^{-1}r$ in samples from a bivariate normal distribution with zero means and zero correlation: [b]15:213, [b]9:61.

Ch(x), also special cases and refs: [17]Nos. 53–5.

$D(\Sigma x_i)$ for $k=3$: Z9:219.

8.22.

$D(x)=\frac{1}{2a}\operatorname{sech}^2\left(\frac{x-m}{a}\right)$, connected with lognormal: [e]12:122.

Called 'logistic', $Ch(x)=\pi x\operatorname{sech}\pi x$ when $m=0$, $a=2$; [v]7:163.

Called 'hyperbolic law of errors', properties, examples: [s]2:55.

Distribution of t and F statistics: *MR 13:665.*

8.23.

Distribution of the correlation ratio, involving series: [a]97:121, [c]24:441.

8.24.

Various distributions of the form exp (−quartic polynomial): [d]4:1, [d]4:79, [d]19:589.

Giving an example where no minimum variance estimator exists: [e]12:43.

MLE: [c]31:188, [a]98:114.

8.25.

$k(1+x^2)^{-m}$: [2]52.

Ch(x): [2]67.

8.26.

$D(x)=b\sin 2(a+bx)$: [b]9:61.

8.27.

Normal multiplied by an eighth degree polynomial: [a]106:361.

8.28.

$D(x)=(\frac{1}{4}h+\frac{1}{4}h^2|x|)e^{-h|x|}$, $D(\bar{x})$: [n]10-3:90.

For $h=1$, $Ch(x)=\left(\frac{1}{1+x^2}\right)^2$: [n]10:75, [17]No. 39.

8.29.

Miscellaneous distributions given in terms of C(x): *[d]13:217, Math. Tables and Other Aids to Computation 5:109, [g]50:209.*

8.30.

$D(x)=\frac{1}{2}k(1+|x|)^{-k-1}$: *[1]225 No. 2.*

k negative: *[c]33:126.*

k=1+p: *[c]36:93, [17] No. 17,* generalization *No. 18.*

8.31.

$D(x)=k \exp(-ax^p)[1+q \sin(bx^p)]$: *[2]106.*

8.32.

Various distributions formed from rational functions of x, rational functions multiplied by $e^{-1/x}$, e^{-x} and $\exp(-\tan^{-1}x)$: *[d]1:137.*

8.33.

$D(x)=(e^2+|x|)^{-1}[\log(e^2+|x|)]^{-2}$, having a pathologically long tail: *[d]17:11.*

8.34. Weibull

$D(x)=ab\ x^{b-1} \exp(-ax^b)$, $x\geq 0$, $a>0$, $b>1$: *[g]43:408, [17]No. 44.*

Moments of order statistics: *[d]26:330.*

8.35.

$D(x)=k \sin^m x \cos^n x$: *[c]30:182.*

n=0, value of k, Ch(cos x): *Phil. Mag. Ser. 7, 39:70.*

8.36.

$D(x)=2h\pi^{-1}(1+h^2x^2)^{-2}$: *[n]10-3:77.*

8.37.

$D(x)=2h\pi^{-1}(e^{hx}+e^{-hx})^{-1}$, D(x) called "Perks": *[n]10-3:90, [d]26:153, [v]7:159, J. Inst. Actuar. 63:12.*

8.38.

$D(x)=(2\pi)^{-\frac{1}{2}}(x/k)[\exp(-\frac{1}{2}(x-k)^2) - \exp(-\frac{1}{2}(x+k)^2)]$: *[2]387.*

8.39.

Four distributions formed by multiplying the normal distribution by a polynomial, used to illustrate kurtosis: *[g]40:259.*

8.40. Extreme Value

$D(x)=a \exp[-a(x-m)] \exp[-\exp-a(x-m)]$: *[d]17:299.*

Gumbel or Fisher-Tippett distribution.

Determination of constants: *C.R. Acad. Sci. Paris 222:34.*

Estimation, MLE: *[d]24:282.*

Bias: *[d]27:758.*

$Ch(x)=\Gamma(1-ixa^{-1})\exp(ixm)$ Cumulants (see reference): *[18]1-144.*

For m=0, a=1, $Ch(x)=\Gamma(1-ix)$, References: *[17]No. 43, [u]24:180.*

Connection with No. 2.3: *[v]4:8, [g]50:518, [g]42:408.*

Special cases: *[r]1:4.*

$\sim D(x)$: *[u]24:180, [g]43:403, Studies presented to Richard von Mises. Academic Press, 1954 p. 346.*

With slight modification $\sim D(x)$ is distribution of log survival time: *J. Hygiene 42:328.*

See also: *U.S. Dept. Agric. ARS 41:13, Ann. Inst. Henri Poincaré 4:115, 5:115, J. de Physique Serie 7 Vol. 8, nos. 8, 11, Bull. Am. Meteor. Soc, 23:95, C.R. Acad. Sci. Paris 246:49, 237: 512, Nature 175:270, Cong. Intl. Math. 1936, 2:200, [a]99:732, [w]8:97 NBS Appl. Math. Ser. No. 33.*

8.41.

$D(x)=\frac{2}{\pi}\frac{x^2}{(1+x^2)^2}$, $Ch(x)=e^{-|x|}(1-|x|)$, *[n]10:75, [17]No.30.*

8.42.

D(x+y), where x and y obey various trivial distributions: *[d]5:16.*

8.43.

$D(x)=Cx^{-1}(1+p/x)^{-2}$: *[d]6:106.*

8.44.

$D(x)=\frac{a+1}{2a}(1-|x|^a)$, $-1<x<1$: *[17]No.15.*

8.45.

$$D(x)=\frac{\lambda^a}{\left(1+\frac{c}{\lambda^b}\right)^n} e^{-\lambda x} x^{a-1} \sum_{j=0}^{n} \binom{n}{j} \frac{c^j x^{bj}}{\Gamma(bj+a)},\quad 0<x<\infty,$$

$\lambda>0$, $c\geq 0$, $b\geq 0$, $a>0$, n=1,2, . . . ,

Ch(x), References: *[17]No. 36.*

8.46.

$D(x)=(x-k)x^n e^{-ax}$: *[g]42:572.*

8.47. Stevens-Fisher

$D(x)=\Sigma \binom{n}{j}(-1)^j(1-jx)^{n-1}$ *[k]9:315, [k]10:14.*

Compare No. 5.16 and No. 8.70.

8.48.

$D(x)=C \exp\ (ax^b-cx)$, $0<x<1$: [d]*25:641*.

8.49.

$D(x)=C \exp[-a(b-x)^{-c}]$: [d]*25:645*.

8.50.

$D(x)=\frac{a}{2\Gamma(1/a)} e^{-|x|^a}$, $-\infty<x<\infty$, $a>0$ [i]*5:133*,[g] *26(Suppl.-H)227*,[17]*No.40*.

MLE: [u]*45:542*.

8.51.

Distribution of non-normal correlation: [c]*38:224*.

8.52.

$D(x)=C\left[\frac{p-x^2}{q+x^2}\right]^{\frac{m}{p+q}}$, $-\sqrt{p}<x<\sqrt{p}$

Value of C, references: [17]*No.31*.

Hansmann's distributions, obtained from a generalized Pearson differential equation: [c]*26:129*.

8.53.

$D(x)=(k/x) \exp[-ax-(b/x)]$: [17]*No.48*.

Called "Type Harmonique": *C.R. Acad Sci. Paris 213:634*.

8.54.

$D(x)=(a/\sqrt{\pi})e^{2a\sqrt{b}} \frac{1}{x^{3/2}} e^{-bx-\frac{a^2}{x}}$: [i]*23:101*.

$Ch(x)=\exp[2a(\sqrt{b}-\sqrt{b-it})]$: [17]*No.50*.

8.55.

$D(x)=\frac{a^p|r|}{\Gamma(p)} x^{rp-1}e^{-ax^r}$, $0<x<\infty$, $a>0$, $p>0$: [17] *No.51*.

8.56.

$D(x)=(a-1)^2 \ (\log\ x)/x^a$, $1<x<\infty$, $1<a$: [17] *No.57*.

8.57.

$D(x)=-[L(b^{1-a})x^a \log\ x]^{-1}$, $b<x<\infty$, $a>1$, $b>1$, and

$L(u)=\int_0^u \frac{dv}{\log\ v}$, $u\geqq 0$: [17]*No.58*.

8.58.

$D(x)=-(a+1)^2x^a \log x$, $0<x<1$, $a>-1$: [17] *No.59*.

8.59.

A generalization of the hypergeometric distribution based on the Whittaker function $x^{m+\frac{1}{2}} e^{-\frac{1}{2}x} {}_1F_1(m+\frac{1}{2}-k, 2m+x; x)$

D(x), Ch(x), references: [17]*No.65*.

8.60.

$D(x)=C\ e^{-(x^2/2a^2)}/(b^2+x^2)$, moments, Ch(x), limiting cases (Cauchy, Normal): [r]*2:293*, [r]*3:139*.

8.61.

$D(x)=(1/\pi)[(1-\cos\ x)/x^2]$: [r]*2:328*.

8.62.

D(x) where $A+Bf(x)$ is (a) normal, or (b) Laplace and f(x) is (i) log x, (ii) log $[x/(1-x)]$, (iii) arcsinh x: [c]*36:149*, [r]*5:283*.

8.63.

$$D(x)=\begin{cases} 1-e^{-\mu NT}, & x=N \\ (1-e^{-\mu T}) \exp[-\mu T(x-1)], & x=N+1, \\ & \qquad N+2, \ldots \end{cases}$$

$N=1, 2, \ldots,\ \mu>0,\ T>0$.

8.64.

Garwood's distribution of length of gaps in traffic: [b]*7:65*, [g]*46:117*, [c]*38:384*.

8.65. Matching

$D(x)=(1/x!)[1-(1/1!)+(1/2!)-\ .\ .\ .\ +(-1)^{n-x}/(n-x)!]$: [a]*118:390*, *MR18:346*.

Generalization: [4]*210*.

8.66.

Cigarette card distribution: [a]*118:391*.

8.67.

Cubic polynomial over a finite range, estimation: [g]*50:196*, [d]*26:505,591*.

8.68.

$$D(x)=\left\{\begin{array}{ll} \frac{mn}{m+n} x^{m-1}, & 0<x<1 \\ \frac{mn}{m+n} x^{-n-1}, & 1\leqq x<\infty \end{array}\right\}$$

Moments, etc.: [g]*50:1137*.

8.69.

$$D(x)=\left\{\begin{array}{ll} mn(m-n)^{-1}x^{n-1}(1-x^{m-n}), & m\neq n \\ n^2x^{n-1}\log(1/x), & m=n \end{array}\right\}$$

Moments: [g]*50:1142*.

8.70.

$D(x)=\frac{n^n}{(n-1)!} \sum_{j\leqq nx} (-1)^j \binom{n}{j}\left(x-\frac{j}{n}\right)^{n-1}$, $0\leqq x\leqq 1$

Compare Nos. 5.16 and 8.47: [d]*26:713*.

8.71.

$D(x)=\frac{(r-1)!\,(r-x)}{x!\,r^{r-x}}$, $x=0, 1, \ldots, (r-1)$

Moments, approximations: *[b]19:339.*

8.72. Arfwedson

$D(x)=\sum_{j=0}^{\infty}(x-j)^n(-1)^j\binom{x}{j}$ *[i]34:121.*

8.73. Stevens-Craig

$D(x)=C\,n^{(x)}\sigma_n^x$, σ_n^x being Stirling's number of second kind: *[k]8:57, [c]40:173.*
Generalization: *[w]7:203.*

8.74.

$D(x)=\frac{1}{4}+x^4$, $-1<x<1$: *[m]6:120.*

8.75. Ising-Stevens

$D(x)=\dfrac{\binom{m-1}{x-1}\binom{m+1}{x}}{\binom{m+n}{m}}$: *Zeit. f. Physik 31:253, [k]9:10, [d]11:370, [t]4:171, [w]8:55.*

8.76.

$D(x)=\frac{1}{\pi\sqrt{m^2-x^2}}$: *Ann. of Math. 27:18.*

8.77.

$D(x)=\frac{x^{-\frac{1}{3}}}{3\sqrt{2\pi}}\exp\left[-\frac{1}{2}(x^{\frac{1}{3}}-b)^2\right]$ *Ann. of Math. 19. 27:*

8.78. Negative Hypergeometric

$D(x)=\binom{n}{x}\frac{B(p+x, q+n-x)}{B(p, q)}$: *[b]10:257, Proc. Int'l. Stat. Conf. Rome 1953 paper 71.*

Obtained by assuming binomial probability to obey Beta.

8.79.

$D(x)=k(1+x)^{-2}$ over various ranges: *[d]22:425.*

8.80.

$D(x)=(n-1)[1-(1/x)^{n-2}](1/x^2)$, $x\geq 1$: *[d]22:425.*

8.81.

$D(x)=-\log x^2/\pi^2(1-x^2)$, $-\infty<x<\infty$: *[d]22:425.*

8.82.

$D(x)=\frac{a^{-(a-bx)^2/2cx}}{\sqrt{2\pi cx^3}}$ Called "inverse Gaussian": *Nature 155:453, [u]43:41, Virginia J. Sci. (new series) 7:160.*

Written

$D(x)=\dfrac{\exp\left(\frac{-c(x-m)^2}{2m^2x}\right)}{\left(\frac{c}{2x^3}\right)^{\frac{1}{2}}}$: *[d]28:362, 696.*

$D(x)=k\,x^n e^{-x^2+ax}$, called "Halphen,": *Publ. Inst. Statist. Univ. Paris 4:38.*

8.83.

$D(x)=a(\alpha,\beta)/[\exp(\alpha^2x^2)-\beta]$: *MR16:381.*

8.84.

$D(x)=a\exp[-k^2\log^2\rho]$ where $\rho=[(x-x_0)(x_2-x_1)]/[(x_2-x)(x_1-x_0)]$: *Z10:313.*

8.85.

$Ch(x)=1/\cosh t$, $t/\sinh t$, $1/\cosh^2 t$: *MR11:443.*

8.86.

$D(x)=2\lambda(\lambda+1)/(x+\lambda-1)(x+\lambda)(x+\lambda+1)$, $\lambda>0$, $x=1,2, \ldots, m=1+\lambda$, $v=\infty$: *[e]18:353.*

8.87.

$D(x)=\dfrac{N!n!}{\prod_{i=0}^{n}x_i!(i!)^{x_i}}$: *[s]5:161.*

8.88.

$D(x)=ce^{cx}/(1+e^{cx})^2$: *[s]1:55, [s]3:133.*

Hyperbolic Error distribution.
A discrete distribution from an urn model: *[d]22:452, [w]7:173.*

8.89.

$D(x)=\frac{e^{-\lambda x}}{\alpha x}\sum_{n=1}^{\infty}\frac{1}{[\Gamma(n)]^2}(\lambda x\alpha e^{-\alpha})^n n^{n-2}$, $0<x<\infty$,

$0<\alpha<1$, $\lambda>0$, $m=\frac{1}{\lambda(1-\alpha)}$, $v=\frac{1-\alpha+\alpha^2}{\lambda^2(1-\alpha)^3}$.

8.90.

$D(x)=k^x\sum_{j=0}^{x-1}\dfrac{\binom{x-1}{j}p^j}{(x-j)!}$, $x=0, 1, \ldots$

Called "Pólya-Aeppli": *[c]40:206.*

8.91.

$D(x)=\frac{1}{x}\binom{2x-2}{x-1}k^{x-1}/(1+k)^{2x-1}$, $x=1, 2, \ldots$

8.92.

$D(x)=\begin{cases}\left(\frac{\lambda}{\lambda+\mu}\right)^N(\mu e^{ax})\Gamma(N, (\lambda+\mu)x)/\Gamma(N), & x>0\\ \left(\frac{\lambda}{\lambda+\mu}\right)^N(\mu e^{ax}), & x<0\end{cases}$

D(y−z) where D(y)=Type III (λ, N), D(z)=Type III (μ, 1)

$$\alpha_k=(-\mu)^{-k}k!\sum_{i=0}^{k}\binom{i+N-1}{i}(-\mu/\lambda)^i$$

$$\kappa_k=(k-1)!\left[\frac{N}{\mu^k}+(-1)^k/\lambda\right]$$

8.93. Miller

$$D(x)=(k+1)\frac{(m+k+1)!}{m!}\frac{(m+x-1)!}{(m+k+x+1)!},\quad x=0,1,\ldots$$

$$\alpha_1=\frac{m+k+1}{k},\quad \sigma^2=\frac{(m+k+1)(k+1)(m+1)}{k^2(k-1)}.$$

Obtained by assuming Pascal parameter to be Beta.

9. Miscellaneous Bivariate

9.1. Cauchy Bivariate

$D(x, y)=(1/\pi)\ [1/(1+x^2+y^2)]^2$: [5]45

$D(x, y)=\frac{1}{2\pi}(1+x^2+y^2)^{-3/2}$: [w]8:235.

9.2. Student Bivariate

$$D(x, y)=(2\pi)^{-1}(1-r^2)^{-\frac{1}{2}}\left(1+\frac{x^2-2rxy+y^2}{n(1-r^2)}\right)^{-\frac{1}{2}(n+2)}$$

Tables: [c]22:408, [c]41:154.
If x,y independent: [3]92.

9.3. Poisson Bivariate

Discussion: [2]136, [f]7:414, [c]39:196, *Psych. Bull. 47:434*, *Proc. Edin. Math. Soc. IIs 4:18*.
Special case obtained from binomial: [i]17:98.

9.4.

Lognormal bivariate: [c]22:130, [d]4:30.

9.5.

Normal-lognormal: [d]4:30.

9.6. Binomial Bivariate

$Ch(x,y)=(a\exp(is+it)+be^{is}+ce^{it}+d)^n$: [2]133.

If $a=0$, $D(x, y)=\frac{k!}{x!\,y!\,(k-x-y)!}p^xq^y(1-p-q)^{k-x-y}$, etc.: [i]17:92, [i]19:209.

See also: [i]36:74, *MR14:995*, *Z18:154*, *MR13:665*.

9.7. Gamma Bivariate

$MGF(x,y)=[(1+s)(1-t)-str^2]^{-p}$: [e]5:140, [c]25:158, *MR3:171*.

9.8. Gamma-Normal

$$MGF(x, y)=(1-s)^{-\frac{1}{2}}\exp\left[\tfrac{1}{2}t^2\left(1+\frac{sr^2}{1-s}\right)\right]:\quad [e]5:144.$$

[c]25:132.

9.9. Hypergeometric Bivariate

$$D(x, y)=\frac{\binom{a}{x}\binom{b}{y}\binom{c}{k-x-y}}{\binom{a+b+c}{k}},$$ various properties:

[i]17:104, [c]16:172, [c]22:140.

Moments: *Ganita 5:97*, *Koninkl. Nederl. Akad (A) 60:124*.

9.10. Negative Binomial Bivariate

Various properties: [i]17:100.

$$D(x, y)=\frac{p^p}{(p+2m)p}\frac{1}{\Gamma(p)}\frac{\Gamma(x+y+p)}{\Gamma(x+1)\Gamma(y+1)}\left[\frac{m}{p+2m}\right]^{x+y}$$

correlation $=\frac{m}{p+m}$, regression etc.: [c]41:79.

Polya-Eggenberger: *MR17:605*.

9.11. Elfving

$D(x,y)=\frac{1}{2}x\exp(-x\cosh y)$, connected with ∼D (range): [c]34:111, [c]36:142.

9.12.

$D(x,y)=C\ e^{-ax-by}\ (1-x+y)^p\ (1+x-y)^q$: [c]14:355.
Rhodes surface: [c]22:134, [c]41:550.

9.13.

$D(x,y)=(1+x/a)^m\ (1+y/b)^n\ [1-(x+y)/c]^n$, Filon-Isserlis surface: [c]15:222, [c]16:180.

9.14.

$D(x,y)=(xy)^k\ (x-y)[(1-x)\ (1-y)]^n$: (c)31:226, [k]9:245.

9.15.

$$D(x,y)=\frac{x^{n-2}(1-y^2)^{\frac{1}{2}(n-4)}}{(1-2rxy+y^2)^{n-1}}:\quad [2]365.$$

9.16.

$$D(x,y)=n^{-2}\frac{n!}{[(k-1)!]^2(n-2k)!}(x/n)^{k-1}(y/n)^{k-1}(1-x/n-y/n)^{n-2k}$$

$x>0$, $y>0$, $x+y<n$, $2k<n$
As $n\to\infty$, x,y→independent Type III(1,k)
Special case, $k=0$: [d]7:149.

9.17.

$D(x,y)=C\ x^{\frac{1}{2}(n-3)}\ (y-x)^{\frac{1}{2}(k-n-2)}\ \exp\ [-\frac{1}{2}(k-1)y]$, $D(x/y)=$Beta: [b]*1:213.*

9.18.

Uniform bivariate, triangular bivariate: [c]*24: 382*, [v]*5:322.*

9.19.

Gram-Charlier bivariate: [c]*36:177.*

9.20.

The fifteen constant surface, (quartic polynomial)$\cdot e^{-Q}$: [c]*17:268.*

9.21.

Pearson's Student-like surfaces: [c]*15:234*, [c]*18: 229*, [c]*22:137.*

9.22.

$D(x,y)=x+y$, $D(x+y)$: [8]*94.*

9.23.

Normal-negative binomial: [k]*13:289.*

9.24.

Edgeworth surface: [c]*38:220*, [c]*17:314.*

9.25.

Rayleigh bivariate: *Electrical Engineering, November 1954, p. 1004.*

9.26.

Discussion of "possible" bivariate distributions, Narumi's system; [c]*15:77, 209, 222.*

Generalization: [c]*22:109.*

9.27. Von Mises-Fisher Distribution

Generalization of No. 8.17: *Proc. Roy. Soc. Lond. Ser. A 217:295*, [c]*43:344.*

9.28.

Beta bivariate: [r]*2:261.*

9.29.

$D(x,y)=k\frac{e^{-\frac{1}{2}(ax^2+2bxy+cy^2)}}{m^2+(ax^2+2bxy+cy^2)}$: [v]*3:153.*

9.30.

$D(x,y)=k\ [1-a^2x^2-b^2y^2+2abrxy]^n$, $n+1>0$, $r^2<1$: [v]*3:273.*

9.31.

$D(x,y)=k\ \exp\ [-Q(x,y)]\cdot[h^2+Q(x,y)]^n$: [r]*3:273*, [v]*5:323.*

9.32.

Defined over (0,0) (0,1) (1,0) from urn model: [r]*3:328.*

9.33.

"Correlation by common factor" surface: [c]*24: 288.*

9.34.

Johnson's system; ten surfaces obtained by translation: [c]*36:297.*

9.35.

Nine surfaces with Pearson or Bessel marginal distributions: *MR5:126*

9.36.

$D(x,y)=C[(x-1)!(h-x)!(y-1)!(k-y)!]^{-1}$: *Hoel, Intro. to Math. Stat. 180.*

9.37.

Type III bivariate, with discussion and calculation of D(r): [c]*7:159.*

9.38.

$D(x,y)=\frac{1}{4}(1+kxy)$, $|k|\leq 1$, $-1\leq x,y\leq 1$: [v]*8:234.*

10. Miscellaneous Multivariate

10.1. Wishart Trivariate

$$D(x,y,z)=\frac{n^{n-1}(xy-z^2)^{\frac{1}{2}(n-4)}}{4\pi\Gamma(n-2)M^{\frac{1}{2}(n-1)}}\exp\left(-\frac{n}{2M}(v_2x-2\mu z+v_1y)\right),$$

$\mu=\rho\sigma_1\sigma_2$, $M=v_1v_2(1-\rho^2)$: [1]*397*, [3]*330*, [4]*226.*

$$Ch(x,y,z)=\left(\frac{A}{A^*}\right)^{\frac{1}{2}n-1}\quad\text{where}\quad A=\begin{vmatrix}\frac{nv_2}{2M} & \frac{\mu n}{2M}\\ -\frac{\mu n}{2M} & \frac{v_1 n}{2M}\end{vmatrix}\ \text{and}$$

$$A^*=\begin{vmatrix}\frac{nv_2}{2M}-is & \frac{\mu n}{2M}-it\\ -\frac{\mu n}{2M}-it & \frac{v_1 n}{2M}-iu\end{vmatrix}.$$

Moments and cumulants: [3]*334.*

As distribution of normal bivariate variance-covariance: [c]*10:510*, [c]*21:164*, [c]*27:230.*

See also: [k]*9:243*, [u] *44:295*, *J. Soc. Stat. Paris 96:262*, [u]*29:264*, [a]*92:580.*

10.2. Wishart Multivariate

$D(x_{ij})=K_{kn}A^{(n-1)}X^{(n-k-2)}\exp(-\Sigma a_{ij}x_{ij})$, where $X=|x_{ij}|$, $A=|a_{ij}|$ and $K_{kn}=\pi^{\frac{1}{4}k(k-1)}[\Gamma(\frac{1}{2}(n-1))\ \ldots\ \Gamma\ (\frac{1}{2}(n-k))]^{-1}$

$Ch(x_{ij})=\left(\frac{A}{A^*}\right)^{\frac{1}{2}(n-1)}$,

where $A^*=|a_{ij}-i\epsilon_{ij}t_{ij}|$ and $\epsilon_{ij}=\begin{cases}1, & i=j\\ \frac{1}{2}, & i\neq j\end{cases}$.

[4]226, [3]331, [1] 391–4, [14]66, [i]30:151, [u]29:260, 271.

Reproductive property: *[4]232.*

Various properties: *[c]20:32, [i]24:185.*

Non-central Wishart: *Proc. Roy. Soc. Lond. Ser. A. 229:364.*

See also: *[d]3:197, [d]15:345, [d]17:409, [d]19:262, [e]3:25, [a]97:120, [c]24:476, [c]36:59, [k]8:244, [c]38:470, [i]36:17, [u]35:336, MR10:387, [b]17:79.*

10.3. Multinomial

$D(x_1, \ldots, x_K)=[n!/\Pi(x_i!)]\Pi(p_ix_i)$: *[6]58, [2]290, [7]124, [18]1–160.*

$MGF(x_1, \ldots, x_K)=(p_ke^{t_1}+\ \ldots\ +p_ke^{t_k})^n$: *[4]51.*

$E(x_i)=np_i$, $Var(x_i)=np_i(1-p_i)$: *[4]52, [14]35.*

Moments: *Bull. Amer. Math. Soc. 41:857*

Introductory article with applications: *[15]36.*

PGF: *[18]1–146.*

Chi-square test: *[e]13:2, [c]36:118, [15]739.*

Information and estimation: *[e]8:325.*

MLE: *[e]18:139.*

Distinguishing between two multinomials, asymptotic form: *[e]7:401.*

Trivariate: *[7]146, [d]21:420.*

Bivariate multinomial: *[d]23:547, Rev. da Fac. de Ciencias de Lisboa 2 Serie (A) 2:197.*

See also: *[d]8:127, [d]21:416, [e]11:367, [d]25:772, [d]28:861, [f]13:451, [t]2:84, Am. Math. Monthly 53:59, Koninkl. Nederl. Akad. (A) 60:121, Z8:122, MR17:56, MR16:839, MR13:665.*

10.4 Type X Multivariate

$D(x_1, \ldots, x_n)=C\ e^{-x/b}$ where $x=\Sigma x_i^2$: *[e]41:54.*

10.5.

Gamma multivariate: *[e]11:45.*

10.6. Student Multivariate

$$D(x_1, \ldots, x_p)=\frac{A^{\frac{1}{2}}\Gamma[\frac{1}{2}(n+p)]}{(n\pi)^{\frac{1}{2}p}\Gamma(\frac{1}{2}n)}\left[1+\frac{1}{n}\Sigma a_{ij}x_ix_j\right]^{-\frac{1}{2}(n+p)},$$

Student for p=1: *[c]41:153, MR16:602.*

See also: *[w]9:143.*

10.7. Cauchy Multivariate

$D(x_1, \ldots, x_n)=C(a^2+x^2)^{-\frac{1}{2}n-\frac{1}{2}}$, where $x^2=\Sigma x_i^2$: *[c]41:54, MR16:51.*

$D(x_1, \ldots, x_n)=C_n\left(1+\sum_{j=1}^{n}x_j^2\right)^{-\frac{1}{2}(n+1)}$: *[w]8:235*

10.8. Spherical

$$D(x_1, \ldots, x_n)=(2\pi)^{-\frac{1}{2}n}\,r^{-\frac{1}{2}n+1}\int_0^\infty \rho^{\frac{1}{2}n}\,J_{\frac{1}{2}n-1}(r\rho)Ch(\rho)d\rho,$$

where $r=\sqrt{\Sigma x_i^2}$, $\rho=\sqrt{\Sigma t_i^2}$: *[c]41:45.*

10.9. Poisson Multivariate

Derivation: *[e]11:120, [d]28:466, [i]37:1.*

Without correlation, multiple Poisson

$$D(x_1, \ldots, x_n)=\exp-(k_k+\ \ldots\ +k_r)\,\frac{k_1^{x_1}\cdot\ \ldots\ \cdot k_n^{x_n}}{x_1!\ \ldots\ x_n!}:$$

[7]127, [e]19:210, Z12:113, 410.

10.10. Binomial Multivariate

$MGF(x_1, \ldots, x_n)=(1+\Sigma p_it_i+\Sigma p_{ij}t_it_j+\ \ldots)^N$: *[e]11:119, Z12:113, 410.*

D(x), etc., in special case: *[i]18:271.*

10.11.

Negative binomial multivariate: *[i]18:274, [i]19:211, Konikl. Nederl. Akad. (A)60:121.*

10.12.

Multinomial multivariate: *[c] 36:47.*

10.13.

Hotelling multivariate: *[k]9:258, [x]7:70.*

10.14.

Multivariate distributions obtained from the normal multivariate: *[i]27:235, [i]28:20.*

10.15.

Generalization of No. 9.14: *[k]9:245.*

10.16.

Generalization of No. 8.3: *[k]11:136.*

10.17.

Generalization of No. 8.6, No. 9.29: [r]*3:153.*

10.18.

Hypergeometric multivariate: [e]*15:391*, [f]*13:488*, *MR17:634*, *MR12:722.*

10.19.

Gram-Charlier multivariate: *MR14:486.*

10.20. **Run Length**

D(x): [d]*11:367*, [4]*202*, *206*, [s]*5:143.*

10.21. **Beta Multivariate**

Tolerance limits: [4]*94.*

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[e] Sankhyā
[f] Biometrics
[g] Journal of the American Statistical Association
[h] Nordisk Statistisk Tidskrift
[h'] Nordic Statistical Journal
[i] Skandinavisk Aktuarietidskrift
[j] Bell System Technical Journal
[k] Annals of Eugenics, Annals of Human Genetics
[l] Econometrica
[m] Applied Statistics
[n] Metron
[o] Annals of the Institute of Statistical Mathematics
[p] Journal of the Institute of Actuaries Students' Society
[q] Bulletin of Mathematical Statistics
[r] Reports of Statistical Application Research, Union of Japanese Scientists and Engineers
[s] Statistica (Neerlandica)
[t] Calcutta Statistical Association Bulletin
[u] Proceedings of the Cambridge Philosophical Society
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V. Appendix. Chronological. Table

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1916	79		11				15														18						
1917	80		11				15																				
1918	81		12				16		1																		
1919	82		12				16		2																		
1920	83		13				17		3					1							19						
1921	84		13				17		4					1							20						
1922	85		14				18	1	5	1				2		2											
1923	86		14–15				18	2	6	2				3							21						
1924	87		16				19	3	7	3				3–4		2											
1925	88		17				20	4	8	4	1			4–5							22						
1926	89		18				21	5	9	5	1			6		2											
1927	90		19				22	6	10	6	2			7		3					23						
1928	91		20A, B				23	7	11	7	3			7		3					24						
1929	92		21				24	1	12	8				8		3					25						
1930	93		22	1			25	2	13	9	4			8		3					26						
1931	94		23	2			26	3	14	10	4			9		3					27						1–2
1932	95		24	3			27	4	15	11	5			9–10		4					28						2–4
1933	96		25	4	1		28		16	12	5	1		10–11		4					29				1		5–7
1934	97	1	26	5	1		29		17	13	6	2		11–12		4					30				2		8–10
1935	98	2	27	6	2		30		18	14	6	3		12		4					31				3		10–12
1936	99	3	28	7	2		31		19	15	7	4				4					32				4		12–15
1937	100	4	29	8	3		32		20	16	7–8	5				5					33				5		15–17
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1939	102	6	31	10	4		34		22	18	9	7				5					35				7		19–21
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1943	106		33	14	6		38		26	22	12	11									39				11	4	
1944	107		33	15	6		39		27	23	12	12									40				12	5	
1945	108		33	16	7	1	40		28	24	12	13				5					41				13	6	
1946	109	8	33	17	7	2	41		29	25	13	14				6			1		42				14	7	
1947	110	9	34	18	8	3	42		30	26	13–14	15				6–7			1	1	43				15	8	
1948	111	10	35	19	9	4	43		31	27	14	16				7–8			2	1	44			1	16	9	
1949	112	11	36	20	9	5	44		32	28	14	17		15	1	8			3	2	45		1		17	10	
1950	113	12	37	21	10	6	45		33	29	15	18			2	9	4		4	2–3	46	1	2	2	18	11	
1951	114	13	38	22	11	7	46		34	30	15–16	19		16	3	10		1	5	3–4	47	2	3	3	19	12	
1952	115	14	39	23	12	8	47		35	31	16–17	20	1		4		5	1–2	6	4	48	3	4	4	20	13	
1953	116	15	40	24	12–13	9	48		36	32	17–18	21	2	17	5	11	5	2–3	7	4–5	49	4	5	5	21	14	
1954	117	16	41	25	13–14	10	49		37	33	18–19	22	3		6	12–13			8	5	50	5	6	6	22	15	
1955	118	17	42	26	14–16	11	50		38	34	19–20	23	4		6–7	13	6	4	9	6	51	6	7	7	23	16	
1956	119	18	43	27	17	12	51		39	35	20–21	24	5		7–8	13	6–7	4	10	6–7	52	7	8	8	24	17	
1957	120	19	44	28	18	13	52		40	36	21	25	6	18	8–9		7	4	11	7	53	8	9		25	18	

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Selected Bibliography of Statistical Literature, 1930 to 1957: IV. Markov Chains and Stochastic Processes

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This is the fourth in a series of bibliographies dealing with various specific subjects in the field of statistics. Titles and references of important contributions to the study of Markov chains and stochastic processes have been taken from technical journals published since 1930 in the many countries that have been actively engaged in statistical analysis.

Two prominent reviewing journals, *Zentralblatt für Mathematik* (1930 to 1940) and *Mathematical Reviews* (1940 to 1957), have provided the source material for this series of bibliographies. The NBS Statistical Engineering Laboratory maintains a current file of abstracts cut from these journals and pasted onto cards, on the subject of probability and mathematical statistics. Each card is coded by subject after the classification system of *Mathematical Reviews*, and filed by author. Authors' names and references are transcribed to punched cards, and classified by subject code to facilitate use of the abstract cards. The listings therefrom, for any chosen subject, when supplied with titles, provide the manuscript for these published bibliographies. The bibliography presented here is on Markov chains and stochastic processes.

Complete coverage of any one of the subjects is, of course, impossible, but the reviewing journals have undoubtedly abstracted the most important publications from a large variety of technical journals and publishing houses. The reviewing journals have generally favored the publications in statistics that deal with theory and methodology; any subclassification on *applications* is consequently very limited.

Following the subject classification of the *Mathematical Reviews Annual Index*, all abstracts that come under the general topic of Markov chains and stochastic processes were consolidated, until 1956, into one category. Since that time, however, the *Index* has subdivided this general group into (A) Stochastic processes; general theory, (B) Markov processes, (C) Stationary processes, (D) Special processes, and we introduced the fifth category, (E) Applications, to include whatever publications on applications that the reviewing journals chose to abstract.

The task of subclassifying the accumulated abstracts from 1930 to 1956 into these finer groupings was accomplished at the Catholic University of America under the general supervision of Professor Eugene Lukacs. Dr. Churchill Eisenhart, Chief of the Statistical Engineering Laboratory, who originally conceived the plans for this project, continues with encouragement and assistance.

The references given here contain the following information taken directly from the abstracts:

Author: The author's surname, followed by initials only. In the case of multiple authorships, the journal reference appears with each author's name, but the title of the paper appears with the senior author only. The symbol ♦ preceding the surname denotes multiple authorship.

Title: Exactly as in the reviewing journal. Titles of separately bound publications (books, reports, theses, etc.) are in italics, followed by the publisher.

Reference to literature: Some very unconventional abbreviations were necessitated by the limited space for transcription to the punched cards. The name of the journal in italics and the number of the volume in bold face, are followed by initial page number.

Date of publication: The next figure, in parentheses, shows the date when the article or book itself appeared.

M (for *Mathematical Reviews*) and *Z* (for *Zentralblatt für Mathematik*) are followed by the volume number and page number of the reviewing journal in which the abstract appears.

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JOURNAL OF RESEARCH of the National Bureau of Standards—B. Mathematics and Mathematical Physics
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Selected Abstracts

Statistical models for component aging experiments, J. R. Rosenblatt, Intern. *Conv. Record. Inst. Radio Engrs.* **8,** *Pt. 6, 115 (1960).*
Statistical methods are considered for analysis of experiments in which electronic components (transistors, say) are subjected to controlled "aging" conditions and measured periodically until "failure". A general framework is proposed for probabilistic models representing the results of such experiments. Some techniques for statistical analysis are outlined which permit simultaneous treatment of variables data and "failure" data, and lead to some possible flexibility in the definition of "failure". The statistical techniques are directed to the problem of estimating the probability of the joint event that (a) the component survives to age T and (b) the value of a component characteristic remains within specified limits.

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